SCALING OF THE SAWTOOTH REPETITION TIME FROM SIMULATIONS WITH REDUCED MHD EQUATIONS, AND COMPARISON WITH EXPERIMENTS IN THE FRASCATI TOKAMAK

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ABSTRACT. The authors propose a scaling of the sawtooth repetition time for Ohmic discharges. This scaling has been obtained from simulations with reduced magnetohydrodynamic equations together with a simple profile consistent transport model. The choice of a profile consistent perpendicular thermal diffusion coefficient allows the scaling of the sawtooth repetition time to be obtained in terms of the diffusion coefficients and the safety factor at the edge, thus generalizing the scaling given by Vlad and Bondeson (Nucl. Fusion **29** (1989) 1139). The scaling is in good agreement with the experimental results for Ohmic discharges in the Frascati Tokamak.

1. INTRODUCTION

Sawtooth oscillations [1], i.e. a periodic sudden drop of the central temperature followed by a slow increase, are commonly observed in tokamaks under a variety of experimental conditions. In 1975, Kadomtsev [2] first proposed a model for explaining sawtooth oscillations. In this model, the drop of the temperature is triggered by the $m = 1/n = 1$ resistive kink mode, which is unstable when the value of the safety factor q in the centre of the plasma column falls below unity (m and n are the azimuthal and toroidal mode numbers). Via resistive reconnection, the helical flux inside the $q = 1$ surface is redistributed in such a way as to produce a symmetrical state with q above unity everywhere and a flat temperature profile in the centre. Then, Ohmic heating leads to a slow temperature increase, which causes peaking of the temperature and current density profiles, making the central q value fall below unity; thus the conditions for a successive internal disruption are established and the cycle is repeated.

Even though Kadomtsev's model is simple, numerical simulations using the cylindrical, low beta, resistive, reduced MHD equations [3] plus an equation for the evolution of the temperature have been very successful in reproducing sawtooth oscillations [4–7]. In particular, Vlad and Bondeson [7] pointed out the importance of including self-consistently computed equilibria to obtain values of the sawtooth period, the precursor growth rate and the crash time which are in agreement with those obtained in small and medium size tokamaks.

Nevertheless, differences between the numerical results presented in Ref. [7] and the experiments have been observed for plasma parameters characterizing large and hot devices (e.g. JET [8]). Actually, in such devices, toroidal, finite beta, ideal effects [9–11] are expected to play an important role.

The use of a very efficient numerical code [6] has permitted an extensive parameter study [7] and has revealed several unexpected and non-trivial dependences of the sawteeth on the transport coefficients. A fixed thermal diffusion coefficient was used in Ref. [7].

We present here a scaling of the sawtooth repetition time in terms of quantities which are easily observed in experiments, and compare this scaling with the experimental data of the Frascati Tokamak (FT). For this purpose, it is necessary to relate explicitly the free parameters used in the simulation code (in particular the thermal diffusion coefficient, which is not directly measured in the experiments) with measured quantities (e.g. the confinement time). The use of a 'profile consistent' thermal diffusion coefficient takes into account the dependence of the temperature profile on the safety factor at the edge, q_a, and permits an explicit relation of the thermal diffusivity with the energy confinement time and with q_a. Such an explicit dependence of the sawtooth repetition time on the energy confinement time and q_a was not considered in Ref. [7].

The scaling obtained using the 'profile consistent' transport model can be very useful in comparing numerical simulations with experimental results of a single device in which q_a and the energy confinement time are well controlled parameters which globally characterize the discharge.

2. THE MODEL

We have used the standard, low beta, resistive, reduced MHD equations, derived in the large aspect ratio, cylindrical limit with $\beta = O(\epsilon^2)$ [3]. An equation for the electron temperature which includes Ohmic heating and highly anisotropic thermal diffusivity is evolved self-consistently. In normalized units, the model equations solved by the code [6] are:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla_\perp\right)\omega = \vec{B}\cdot\nabla j + \nu\nabla_\perp^2\omega$$

$$\frac{\partial\psi}{\partial t} = \vec{B}\cdot\nabla\phi - \eta j + E_z(t) \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla_\perp\right)T = \frac{2\eta}{3} j^2 + \nabla_\perp\cdot(\chi_\perp\nabla_\perp T) + (\vec{B}\cdot\nabla)\chi_\parallel(\vec{B}\cdot\nabla)T$$

where ψ and ϕ are, respectively, the magnetic flux and stream functions, $\vec{B} = \nabla\psi \times \hat{z} + B_T\hat{z}$, $\vec{v} = \nabla\phi \times \hat{z}$, $\omega = -\nabla_\perp^2\phi$ is the vorticity, $j = -\nabla_\perp^2\psi$ is the plasma current, T is the electron temperature, η is the Spitzer resistivity and $\nabla_\perp = \nabla - \hat{z}\ \partial/\partial z$. The lengths are normalized to the minor radius a and the times are normalized to the Alfvén transit time, $\tau_A = R/v_A$, where R is the major radius and v_A is the Alfvén velocity in the toroidal magnetic field B_T. ν is the perpendicular viscosity and $\chi_\perp$ is the perpendicular thermal diffusivity, which are both multiplied by τ_A/a^2, and $\chi_\parallel$ is the parallel thermal diffusivity, which is multiplied by τ_A/R^2. Mass density, viscosity and parallel thermal conductivity are taken to be constant in space and time. In normalized units, $T = (R/a)^2\beta/2$; the resistivity is given in terms of the inverse of the Lundquist number $S = \tau_R/\tau_A$, where $\tau_R = a^2\mu_0/\eta$ is the resistive time. The toroidal electric field $E_z(t)$ is adjusted in time such that q_a is kept fixed. The code uses finite differences in the radial direction and Fourier expansion in the azimuthal and toroidal directions. We have considered 150 non-equidistant radial points and single-helicity perturbations with m/n = 1 and mode numbers up to m = 4.

It is well known that the radial profiles in tokamaks depend on q_a. In particular, in Ohmic discharges, more peaked temperature profiles are usually observed when q_a is increased. To take into account this feature, a profile consistent perpendicular thermal diffusion coefficient has been assumed [12]. Thus, assigning a radial temperature dependence of the form

$$T(r) = T_0 \exp\left(-\alpha_q\left(\frac{r}{a}\right)^2\right) \tag{2}$$

with $\alpha_q = \frac{2}{3}q_a$, and balancing the Ohmic input and the perpendicular thermal diffusion term in the equilibrium equation for the electron temperature, we obtain

$$\chi_\perp(r) = \chi_{\perp 0}F\left(\frac{\alpha_q}{2}\left(\frac{r}{a}\right)^2\right)\frac{n_0}{n}$$
$$F(x) = \frac{e^{2x} - e^{-x}}{3x} \tag{3}$$

Here, the function of the parameter α_q is to renormalize the effective width of the current channel as q_a is varied. With this choice of the perpendicular thermal diffusion $\chi_\perp(r)$, the resonant surface $r_1 = r(q = 1)$ will change as q_a is varied. Thus, in the following, the scaling of the sawtooth repetition time is given in terms of the diffusion coefficients at the resonant surface r_1 (although these values will not differ much from the values in the centre because of the average flattening of the profiles due to sawtoothing). The parallel thermal diffusivity has been kept fixed at $\chi_\parallel = 26.67$; in the simulations discussed below, this prevents the formation of a steady m = 1/n = 1 convection pattern that completely eliminates sawteeth [5, 6].

3. SCALING OF THE SAWTOOTH REPETITION TIME

In Ref. [7] it is clearly pointed out that a necessary condition for distinct relaxation to occur is that the perpendicular viscosity must be comparable to, or larger than, the perpendicular thermal diffusivity, $\nu/\chi_\perp \geq 1$. Thus, ν and $\chi_\perp$ will be varied in the following such that the requirement $\nu/\chi_\perp \geq 1$ is fulfilled. Some differences with respect to the scaling given in Ref. [7] can be expected, because of the different model used for $\chi_\perp$.

The scaling of the sawtooth repetition time τ_{saw} with ν and $\chi_\perp$ has been performed prescribing $q_a = 3$ and $S(\hat{r}_1) = 2.3 \times 10^5$, i.e. with a moderate value of the Lundquist number (to be compared with a typical value for the FT tokamak of $S \approx 10^7$).

For fixed $\chi_\perp(r_1) = 3 \times 10^{-5}$, we obtain $\tau_{saw} \propto \nu^{0.2}$, whereas, for fixed $\nu = 4.25 \times 10^{-5}$, a best fit analysis gives $\tau_{saw} \propto \chi_\perp^{-\alpha}$, with $\alpha = 0.5$ and an estimate of the error on the exponent $\Delta\alpha = \pm 0.1$; thus, $\tau_{saw} \propto \chi_\perp^{-0.5\pm0.1}$ (Fig. 1(a, b)). The same assumptions as in Ref. [7] have been made for the scaling of τ_{saw} with S: The perpendicular viscosity has been taken to be of the same order as the thermal diffusivity coefficient, $\nu \approx \chi_\perp$, as suggested by experimental results concerning momentum confinement [13–16]. Moreover, $\chi_\perp S$ (and thus νS) has been held constant (the constancy of $\chi_\perp S$ implies that the normalized temperature remains essentially unchanged as S is varied, thus keeping fixed the ratio of the electron energy confinement time, $\tau_E = 3nT/2\eta j^2$, to the resistive time τ_R ($T \propto \tau_E/\tau_R$)). Assuming $q_a = 3$, $\chi_\perp(r_1)S(r_1) \approx 7$, $\nu(r_1)S(r_1) \approx 10$, it is found that $\tau_{saw} \propto S^{0.75}$ (Fig. 1(c)).

The scaling of τ_{saw} with q_a has been performed with $S(r_1) = 2.3 \times 10^5$, $\chi_\perp(r_1)S(r_1) \approx 7$, $\nu(r_1)S(r_1) \approx 10$, and varying q_a from 2 to 5. The dependence of the

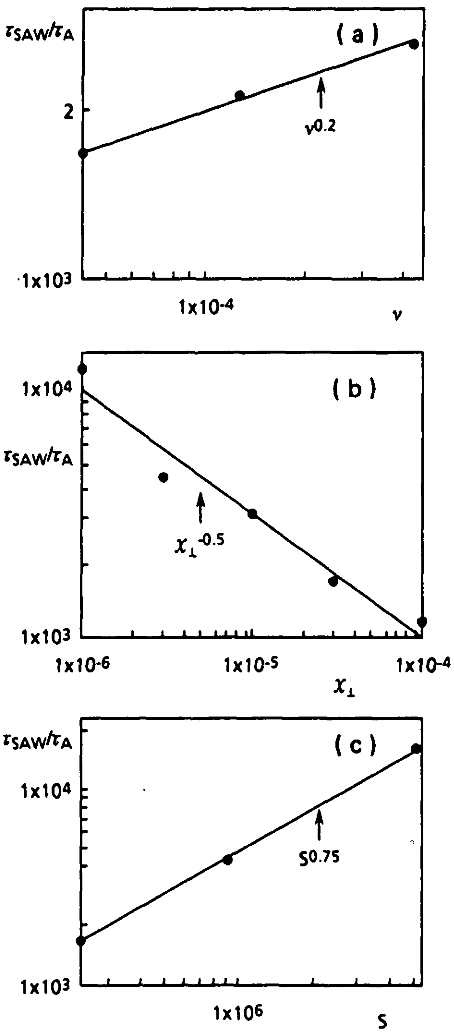

FIG. 1. Sawtooth repetition time τ_{saw}/τ_A versus: (a) perpendicular viscosity ν, (b) perpendicular thermal diffusivity $\chi_\perp$ and (c) Lundquist number S. The values of the other parameters are defined in the text.

q = 1 radius r_1 on q_a follows closely the experimentally observed relation $r_1 \propto 1/q_a$ (Fig. 2(a)), and the sawtooth period is found to scale as $\tau_{saw} \propto q_a^{-0.8}$ (Fig. 2(b)).

Combining the above mentioned parametric dependences, we can give the following numerically obtained scaling for the sawtooth repetition time:

$$\tau_{saw} = k\chi_\perp^{-0.5}\nu^{0.2}S^{0.45}q_a^{-0.8} \tag{4}$$

with k being a numerical constant (k ≈ 0.8).

4. COMPARISON WITH EXPERIMENTAL RESULTS FROM FT

It will be useful to express the scaling (4) in terms of quantities that are routinely obtained from experiments in FT. Unfolding the normalizations used in the code and assuming $\nu \sim \chi_\perp$, Eq. (4) becomes

$$\tau_{saw} = k\left(\frac{a^2}{\chi_\perp\tau_A}\right)^{0.3}\left(\frac{\tau_R}{\tau_A}\right)^{0.45} q_a^{-0.8}\tau_A \tag{5}$$

With the assumed profile consistent model and from the definition of the global energy confinement time τ_E it is possible to relate the constant $\chi_{\perp 0}$ appearing in Eq. (3) to τ_E:

$$\chi_{\perp 0} = \frac{9}{8}\frac{a^2}{\tau_E q_a} f \tag{6}$$

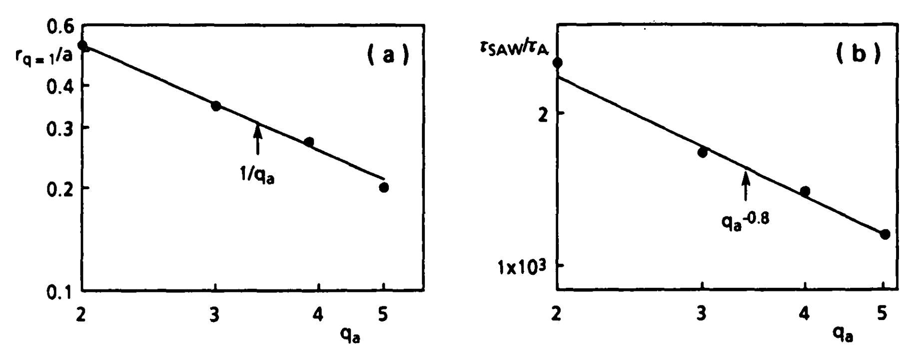

FIG. 2. (a) q = 1 radius and (b) sawtooth repetition time τ_{saw}/τ_A, versus q_a. The values of the other parameters are defined in the text.

with f being a profile averaging factor. For a constant density profile, we have

$$f = \tfrac{3}{2}(1 - e^{-2q_a/3})/(1 - e^{-q_a})$$

and for a density profile of the form

$$n \sim \left(1 - \frac{r^2}{a^2}\right)^{\alpha_n}$$

with $\alpha_n = 1$, we have

$$f = \frac{3}{2}\left(1 - \frac{3}{2q_a} + \frac{3}{2q_a} e^{-2q_a/3}\right)\Big/(1 - e^{-q_a})$$

Also the temperature which enters in the definition of τ_R can be expressed in terms of τ_E:

$$T(r_1) \sim \left(\frac{\tau_E B_T^2 \ln\Lambda}{g n_0 R^2 \Lambda_{Z_{eff}}}\right)^{2/5}\left(1 - \frac{r_1^2}{a^2}\right)^{\alpha_T} \qquad (7)$$

where n_0 is the electron density at the centre, $\ln\Lambda$ is the Coulomb logarithm and $\Lambda_{Z_{eff}}$ is the Z_{eff} dependence of the resistivity [17], $\Lambda_{Z_{eff}} = (3.4/Z_{eff})(1.13 + Z_{eff})/(2.67 + Z_{eff})$. The profile averaging factor g is related to the experimental profiles; in the following, it is assumed that $j(r) \sim \exp(-q_a(r/a)^2)$ (as assumed by the profile consistent model), and the radial profiles of n and T are represented by generalized parabolas, with exponents α_n and α_T, respectively; thus $g = q_a/[(1 - e^{-q_a}) \times (\alpha_n + \alpha_T + 1)]$.

Evaluating $\chi_\perp$ and τ_R at $r = r_1$, the scaling (5) becomes

$$\tau_{saw}[s] = 0.11 \frac{\left(1 - \frac{1}{q_a^2}\right)^{0.3 + 0.45q_a}}{\left[F\left(\frac{1}{3q_a}\right)\right]^{0.3} f^{0.3} g^{0.27}} \times \frac{\tau_E^{0.57}[s]\, a^{0.9}[m]\, \Lambda_{Z_{eff}}^{0.18}\left(\frac{15}{\ln\Lambda}\right)^{0.18} B_T^{0.29}[T]\, \mu^{0.125}}{q_a^{0.5}\, R^{0.29}[m]\, \langle n\rangle^{0.145}[10^{20}\ m^{-3}]} \qquad (8)$$

where we have assumed $\alpha_n = 1$, $\alpha_T = \frac{2}{3}q_a$ (thus, the line averaged density is $\langle n\rangle = 2n_0/3$), $r_1/a \approx 1/q_a$, and μ is the ratio of ion mass to proton mass.

Figure 3 is a plot of the experimental sawtooth repetition time versus the proposed scaling. 299 shots of hydrogen, deuterium and helium gases are included, with $2.2 \le q_a \le 5$, $1.8 \le \tau_E[ms] \le 40$, $2.4 \le B_T[T] \le 8$, $0.1 \le \langle n\rangle[10^{20}\ m^{-3}] \le 3$, $a = 0.20$ m, $R = 0.83$ m. The agreement is good with a linear correlation coefficient $r = 0.77$ (the scaling proposed in Eq. (2) of Ref. [7], applied to the same database, gives correlation coefficients $0.43 \le r \le 0.63$, depending on whether the central electron temperature or the average electron temperature is used). It is emphasized that the proposed expression (8) does not contain any adjustable coefficients; thus, the absolute value of the sawtooth repetition time is predicted. The major uncertainty in the derivation of the scaling (as discussed in Section 3) arises from the exponent of $\chi_\perp$ and thus from the exponent of τ_E in Eq. (8). This leads to to an error of the predicted sawtooth repetition time which is estimated to be of the order of 35%. Also the experimental data are affected by errors; in particular, the density profile on FT was not available for all discharges [18], which also influences the evaluation of the energy confinement time. Thus, the error arising from the experimental data in evaluating Eq. (8) is estimated to be of the order of 30%.

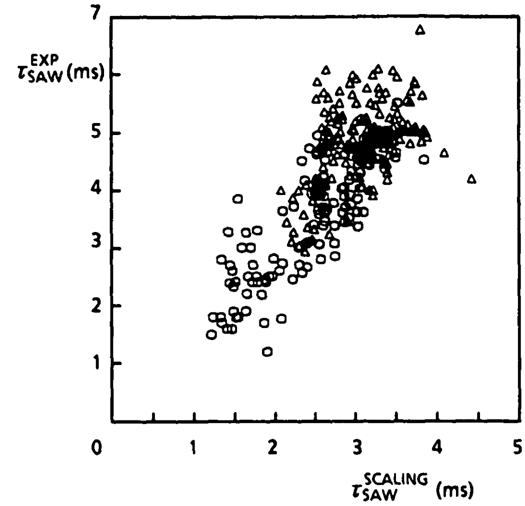

FIG. 3. Exprimental sawtooth repetition time versus the scaling given by Eq. (8). Included are 299 shots of hydrogen, deuterium and helium gases; the linear correlation coefficient is r = 0.77. The open circles are data for 'non-saturated' discharges.

Note that the data presented in Fig. 3 include both discharges with a linear dependence of τ_E on density ('non-saturated' discharges) and discharges with a very weak dependence of τ_E on density ('saturated' discharges) [18]. This strongly suggests that a transition between two different regimes occurs. Any discussion of the nature of these confinement regimes is beyond the scope of this letter; however, the good agreement of scaling (8) shown in Fig. 3 indicates that the energy confinement time contains sufficient information on the transport to correctly describe the sawtooth repetition time. This leads to the conclusion that the reduced, resistive MHD equations together with a simple profile consistent transport model are adequate for estimating the sawtooth repetition time in small and medium size tokamaks.

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STATIONARY EFFECT OF ENERGETIC TRAPPED PARTICLES ON THE RESONANT EXCITATION OF THE IDEAL BALLOONING MODE

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ABSTRACT. The effect of the contribution of zero frequency (stationary) energetic trapped particles on magnetohydrodynamic modes and the resonant excitation of these particles is studied on the basis of the dispersion relation which is applicable to both the internal kink mode and the ballooning mode in tokamaks. The stability window is found to be significantly enlarged by the steady contribution of energetic trapped particles.

The interaction of energetic trapped particles with the internal kink mode and their resonant excitation near the precessional drift frequency ω_d was studied [1] in order to interpret fishbone oscillations observed during neutral beam injection (NBI) in tokamaks. The theory was applied to realistic cases, including stabilizing effects such as the ion diamagnetic effect of the bulk plasma, to find an explanation for the suppression of sawtooth oscillations during injection of radiofrequency (RF) waves and NBI in JET [2-5].

The contribution of energetic trapped particles consists of a zero frequency steady component and a kinetic resonant component which vanishes at the zero frequency limit. Rosenbluth et al. [6] theoretically predicted that the steady state component can effectively stabilize the