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FREE BOUNDARY TOROIDAL STABILITY
OF IDEAL AND RESISTIVE INTERNAL KINKS

G. VLAD. Associazione Euratom ENEA sulla Fusione, C.R.E. Frascati
C.P. 65-00044 -Frascati, Rome, Italy

H. LUTJENS, A. BONDESON, Centre de Recherches en Physique des Plasmas,
Association Euratom - Confédération Suisse, École Polytechnique Fédérale de Lausanne,
21 Av. des Blancs, CH-1007 Lausanne, Switzerland

I. INTRODUCTION

Recently, there has been a strong interest in the stability properties of the internal kink mode. This has been spurred mainly by experimental observations of sawtooth oscillations on large tokamaks revealing unexpected features such as double sawteeth with partial reconvergence, fast crashes, and central q-values well below unity [1]. These observations are all difficult to reconcile with the conventional Kadomtsev model in which the crash is triggered by the resistive kink mode becoming unstable when the safety factor q falls below unity. The theoretical understanding of the sawtooth activity is made difficult by the sensitivity of the internal kink mode to several factors such as q-profile, pressure, resistivity, aspect ratio, shaping of the cross-section, and even wall position. In addition, the internal kink in a torus (with q > 1/2) is a rather weak instability and should therefore be sensitive to kinetic effects.

Here, we present results from a study of the resistive and ideal MHD properties of the internal kink mode. Generally, we have numerically computed growth rates as functions of various parameters using the full-resistive-MHD toroidal stability code MARS [2] and the cubic spline equilibrium code CHEASE [3].

II. INFLUENCE OF THE CURRENT PROFILE

A major uncertainty in the understanding of the sawtooth is the shape of the current profile. Here, we restrict attention to circular cross-section with an aspect ratio of 4 and choose two types of current profiles. One has uniformly low shear inside a certain radius r = r_s = 0.4, outside which the shear increases rapidly. The other has shoulders in the current profile which produce locally low shear near q = 1, while the central safety factor q_0 is well below unity. We refer to these as "low-shear" and "TEXTOR" profiles [1], respectively.

Figure 1 shows the low-shear current profile j = j_{ls}(r) (where l is a flux surface label proportional to the square root of the enclosed volume). This current profile is monotone, but the shear is q = (q_0 + q_r)/2, which has a slight local minimum at r = r_s = 0.4, with q(r_s) = 0.03. The shear does not exceed 0.052 in the entire region r < r_s. Profiles of this type may arise in sawtoothing plasmas if total reconvergence occurs within the q = 1 region, r < r_s, followed by neoclassical peaking of the current during the rise phase [4]. We have considered a family of equilibria with self-similar current profiles where we specify the q-value at radius r = r_s. For this family, the central safety factor q_0 is related to q_0 = q_0(r) by q_0 = 0.94q_0. It may be useful to think of these equilibria as an approximation to the sequence in time during the ramp phase of a sawtooth, with q_0 and q_0 decreasing functions of time. Figure 2 shows the resulting growth rates for four different values of poloidal beta at the q = 1 surface (0.00, 0.05, 0.10, and 0.15) at Lundquist number \( S = \tau_{m}/\tau_{H} = 10^6 \) (Fig 2a), 10^7 (2b) and 10^8 (2c). For this q-profile, q_0 = 1 locates the minimum shear (s = 0.02) at the q = 1 surface, and when q_0 is decreased below unity, the q = 1 surface moves out into the region of high shear. For instance, q_0 = 0.98 gives \( \delta(q=1) = 0.22 \). In all cases shown in Fig. 2, a conducting wall is assumed at r = b = 1.2a.

We note from Fig. 2 that complete resistive MHD stability is very rarely achieved. However, in many cases, the resistive growth rates are small, and we are led to the conclusion that a weak internal kink is stabilized for most of the sawtooth cycle by effects not included in the model. A likely candidate for such stabilization is diamagnetic rotation. In present day tokamaks, \( \omega_{pe}/\omega_{ci} \) is typically between \( 0.3 \times 10^{-3} \) and \( 10^{-4} \), which is comparable to resistive...
MHD growth-rates of the internal kink. It therefore appears plausible that diamagnetic rotation can stabilize the internal kink as long as its resistive-MHD growth-rate is not too large.

By comparing Figs. 2a-c for different values of $S$, we note a gradual change in the main factor determining the growth-rate. At $S = 10^2$, the growth-rates are mainly dependent on $q_p$, i.e., on the shear at the $q = 1$ surface, and are only weakly dependent on the pressure. This is typical of the resistive kink mode. This picture is modified somewhat at $S = 10^8$ which represents an intermediate case. The three lower curves in Fig. 2b ($\beta_{pol} = 0.00, 0.05, 0.10$) still show reasonably high growth-rates (about $2 \times 10^{-3 \Delta A}$) for a current driven resistive mode, but for $\beta_{pol} \leq 0.1$, the pressure has only a weak influence on the growth-rates.

For $\beta_{pol} = 0.15$, destabilization by pressure becomes significant. An interesting aspect of this "pressure driven" instability is that it is clearly sensitive to the $q$-profile and its growth-rate rises sharply when the $q = 1$ surface moves out into the region of high shear. Thus, although the instability is pressure driven, it can be thought of as being triggered by the current profile.

Figure 2a shows the growth-rates for low-shear current profile and different $\beta_{pol}$ values. Much more clear changes from the low-$S$ picture are seen in Fig. 2c for $S = 10^{10}$. Here, the resistive growth-rates for $\beta_{pol} = 0.00$, and 0.05 are rather small (a few times $10^{-4 \Delta A}$) and the pressure driven instability is clearly dominant. Comparing the growth-rates with those for $S = 10^8$, we see that the pressure driven instability is essentially ideal. Thus, it appears that as $S$ is increased, the linear instability leading to the sawtooth crash becomes more and more an ideal, pressure driven instability. However, this ideal mode is sensitive to details of the current profile and can be triggered by a slight shift in the $q$-profile. In fact, the variation of the

Figure 3. TEXTOR current profile

Figure 4. Resistive growth-rates for TEXTOR current profile and different $\beta_{pol}$ values

We have also computed resistive growth-rates for a current profile of TEXTOR type, as shown in Fig. 3. The shoulders in the current profile were adjusted so that the shear has a minimum of about 0.034 at $r = r_p = 0.44a$ in this case, the global shear in the central region is strong and $q_p = 0.634$, is well below unity. Figure 4 shows the growth-rates for different $S$-values and $\beta_{pol} = 0.0, 0.1, 0.2$, and 0.3. In general terms, the behavior is similar to that for the low-shear profile, but the TEXTOR profile supports about twice the pressure before becoming ideally unstable.
III. WALL AND SHAPING EFFECTS

Although the displacement of the internal kink mode is mainly localized to the region inside the \( q = 1 \) surface, the magnetic perturbation outside \( q = 1 \) is not small and it is important for the mode stability [3]. Numerically, we find that the ideal stability boundaries are strongly influenced by wall position when the \( q = 1 \) radius is sufficiently large and the aspect ratio is low. As an example, Fig. 3a shows ideal growth-rates for the ideal internal kink as a function of \( \psi = \beta_0 / \alpha = 0.0 \) and edge \( \beta \) between 2 and 3. Two different wall positions have been considered: \( \beta_0/\alpha = 1 \) (free boundary) and \( \beta_0/\alpha = 2 \) (free boundary). For this equilibrium, we find that the difference in marginal \( \beta_0/\alpha \) between the fit and free boundary cases scales as the square of the inverse aspect ratio, as expected from large aspect ratio theory. At low aspect ratio, the wall position plays a small role, the marginal \( \beta_0/\alpha \) for free boundary stability is only about half of the fixed boundary value for \( \beta_0/\alpha = 2.7 \). When the edge \( \beta_0 \) is raised above 2 for the circular equilibrium, the influence of the wall position is weak and of little practical significance.

In the case of a shaped cross-section and a large \( q = 1 \) surface, the wall position has a more significant influence. As is shown in Fig. 3b for staggered cross-section with elongation \( \kappa = 1.7 \) and triangularity \( \delta = 0.3 \). The aspect ratios are \( \kappa = 3.3, 6, \) and 10, respectively, and \( \beta_0/\alpha = 0.66, 0.7 \) and 0.7 are held fixed. The edge \( \beta_0 \) varies with aspect ratio but remains between 3 and 4. For this equilibrium, the wall has an important influence independent of the aspect ratio. Note that for \( \kappa > 3 \), the equilibrium is free-boundary unstable even at zero \( \beta_0 \). This is in agreement with large aspect ratio theory, as \( \kappa W \) contains stabilizing terms \( \kappa^{-2} \) and destabilizing terms \( \kappa^{-1} \). Thus, at fixed \( \kappa > 1 \), large aspect ratio equilibrium becomes unstable even at zero \( \beta_0 \) when the aspect ratio is sufficiently large. The larger \( \kappa \) has a stabilizing influence, but for JET geometry and the \( q \)-profiles used here, this stabilization is insufficient to compensate for the destabilization by ellipticity.

![Figure 5. Ideal growth-rates for free and fixed boundary at different aspect ratios. Free boundary indicated by open symbols. (a) Circular and (b) JET cross-section.](image)

**REFERENCES**


**COMPUTING THE DAMPING AND DESTABILIZATION OF GLOBAL ALFVÉN WAVES IN TOKAMAKS**

W. Keften, S. Poole*, J.P. Goodblood**, G.T.A. Hayden**, B. Keegan, and E. Schwartz*

* Max-Planck Institut für Plasmaphysik, Garching, Germany  
** FOM-Instituut voor Plasmafysica, Nieuwegein, The Netherlands

The role of ideal MHD in magnetic fusion is in the first place to discover magnetic geometries with favourable equilibrium and stability properties. Non-ideal effects cause slower and weaker instabilities leading to enhanced transport and often to violent disruptions.

MHD spectroscopy, i.e. the identification of ideal and dissipative MHD modes for the purpose of diagnosing tokamaks and optimizing their stability properties, requires a numerical tool which accurately calculates the dissipative MHD spectra for measured equilibria. The new spectral code CASTOR (Complex Alfvén Spectrum for TORoidal Plasmas), together with the equilibrium solver HELENA [6], provides such a tool. In CASTOR, the fluid variables \( p, v, T \), and \( \beta \) are discretized by means of a combination of cubic Hermite and quadratic finite elements for the radial direction and Fourier modes for the poloidal coordinate. The equilibrium is non-orthogonal flux coordinates \( \psi, \xi \), with straight field lines is computed using isoparametric bicubic Hermite elements, resulting in a very accurate representation of the metric elements. Finally, for analysis of JET discharges the equilibrium solver HELENA is interfaced with the equilibrium identification code IDENTIC(D).

All quantities are expanded around an axisymmetric equilibrium \( \Theta = 0 \) of the form

\[
f_{r,\rho}(s,\theta) = f_{\rho}(s,\theta) + \phi(s) f_{\theta}(s,\theta), \quad \text{with} \quad s = \sqrt{\rho v_x},
\]

(1)

Here, \( \lambda \) is the eigenvalue. The imaginary part of \( \lambda \) corresponds to oscillatory behaviour, while a negative real part yields damping and a positive real part yields an exponentially growing instability. With resistivity \( \eta \), the equations for the perturbed density \( p \), velocity \( v \), temperature \( T \), and vector potential \( A \) in normalized units read

\[
\lambda \rho = -v \cdot (\rho_0 v),
\]

(2a)

\[
\lambda \rho_0 v = -\nabla (\rho_0 T_0) + (\nabla \times B_0) \times b + (\nabla \times b) \times B_0 - \nabla \cdot \Pi,
\]

(2b)

\[
\lambda \rho_0 T = -\rho_0 v \cdot \nabla T_0 - (\gamma - 1) \rho_0 T_0 \nabla \cdot v,
\]

(2c)

\[
\lambda A = v \times B_0 - \eta \frac{\nabla \times B_0}{B_0}, \quad \text{where} \quad \eta = \eta v \times A.
\]

(2d)

The pressure tensor \( \Pi \) contains the influence of the anisotropic bulk plasma and of the energetic ions interacting with the fluid. The latter requires the solution of the linearized Vlasov equation for both trapped and passing particles yielding a complicated dependence on \( \lambda \).