Electron fishbone simulations in tokamak equilibria using XHMGC

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Abstract
This paper discusses the first nonlinear numerical simulation results of fishbone excitation by a magnetically trapped supra-thermal electron population with pressure profile peaked on axis due to, e.g. electron cyclotron resonance heating. The precession resonance underlying the linear instability is clearly identified. Meanwhile, mode saturation is shown to occur because of the secular resonant particle motion, pumping particles out of the radial region where the wave–particle power exchange is initially localized. The mode nonlinear saturation is accompanied by downward frequency chirping, due to phase-locking of the mode with resonant particles, consistent with the ‘mode particle pumping’ mechanism.

Some figures may appear in colour only in the online journal

1. Introduction

Internal kink instabilities exhibiting fishbone like behaviours have been observed in a variety of experiments, where a high-energy electron population was present, due to, e.g. electron cyclotron resonance heating (ECRH) [1–4] and/or lower hybrid heating/current drive (LHH/LHCD) [5–7]. These electron fishbones have linear dispersion relation and excitation mechanisms that are very similar to those of energetic ion driven fishbones [8, 9]. Moreover, they are characterized by a very small ratio between the resonant particle orbit width and the extension of the plasma region affected by the rigid radial kink-type displacement, which can also be considered as the characteristic fishbone length scale. This feature is also expected to characterize ion fishbones in burning plasmas of fusion interest due to the large plasma current, while it is not realized for the energetic ions in present-day experiments. Meanwhile, apart for the ratio between resonant particle orbit width to the characteristic fishbone scale length, fluctuation induced transport of magnetically trapped resonant particles, due to precession resonance, is expected to depend on energy and not mass because of the bounce averaged dynamic response [5]. These analogies between electron fishbones in present-day devices and fishbones in burning plasmas make the investigation of the former a good basis to study and predict the effects of the latter.

The first experimental observations of electron fishbones refer to modes propagating in the ion diamagnetic direction, in the presence of inverted pressure profiles (peaked off axis) [1, 3]. Linear theory allows us to interpret these observations as the destabilization of modes by precession resonance with supra-thermal electrons close to the trapped-passing boundary and characterized by toroidal precession reversal. In fact, modes propagating in the ion diamagnetic direction can minimize continuum damping, due to the asymmetry of the shear Alfvén continuous spectrum near the ion diamagnetic frequency gap [5]. Theory also predicts the possibility of exciting, above a higher energetic particle threshold, electron fishbones propagating in the electron diamagnetic direction with on-axis peaked pressure profiles, via precession resonance with deeply trapped particles. The asymmetry of the threshold with respect to the propagation direction should disappear for sufficiently high fast particle energies, corresponding to large enough resonance frequencies compared with the thermal ion diamagnetic frequency [10]. The destabilization of ion fishbones by transit resonance with circulating ions has also been investigated [11, 12]; such destabilizing mechanism for electron fishbones should, however, rapidly lose its efficiency in the velocity space region away from the trapped-passing boundary [13].

While much effort has been devoted to the investigation of the electron fishbone linear physics, the nonlinear physics has been less explored: limited experimental evidence is indeed available of electron fishbone mode structure evolution [14] and energetic electron redistribution [4]. On the theoretical
side it has been suggested [5, 15] that trapped supra-thermal electron transport induced by fishbones is dominated by nonlinear $E \times B$ fluxes, as magnetic flux transfer is bounce averaged to zero. Therefore, it is apparent that the comprehension/prediction of these phenomena would greatly benefit from a numerical simulation approach. Until now, however, self-consistent numerical simulations have been performed only for the ion fishbone mode [16], by an extended/hybrid magnetohydrodynamic (MHD)-gyrokinetic code [17].

The analysis of [16] demonstrated numerically for the first time that the nonlinear fishbone burst cycle occurs because of rapid radial redistribution of energetic particles during the frequency chirping phase of the mode, which eventually yields saturation of the fluctuation intensity, which then finally decays. In this work, we want to apply the self-consistent nonlinear numerical simulation approach to electron fishbones [18, 19], adopting the extended/hybrid MHD-gyrokinetic code XHMGC, described in [20]. With respect to [16], addressing electron fishbone excitation poses the numerical challenge of properly handling the effective bounce averaging of linear and nonlinear mode dynamics, due to the extremely fast parallel electron motion along equilibrium magnetic field lines, which has been addressed and solved by sub-cycling in the particle-in-cell numerical simulation scheme.

In order to simplify the physical processes involved in our simulations, we address electron fishbone excitation by a population of magnetically trapped supra-thermal electrons with the pressure profile peaked on axis, generated, e.g. by ECRH. In this way, we do not directly address typical experimental conditions for electron fishbone excitation (see e.g. the recent [14]) but, at the same time, we avoid the twists in the underlying physics and relative interpretation, connected with the varying mix of circulating versus magnetically trapped particles [13], and the changing of the MHD equilibrium potentially produced by circulating particles [5]. This simplified model bears, anyway, the essential physics ingredients to describe experiments characterized by an anisotropic supra-thermal electron distribution function peaked on axis, produced by strong on axis ECRH and/or combined central ECRH/LHCD (synergy) [21, 22] (as, e.g. planned in future Frascati Tokamak Upgrade (FTU) experimental campaigns). Furthermore, the numerical simulation results, reported here, allow for the first time the identification of the physics mechanism underlying (secular) resonant particle transport and mode frequency chirping for electron fishbones, showing that it is the same conjectured for ion fishbones [9, 23]. Meanwhile, those reported in this work are the first nonlinear self-consistent numerical simulations, which take into account the kinetic response of thermal plasma ions, which is important for a proper treatment of effective plasma inertia and ion Landau damping damping [5, 10, 24, 25].

Similar to ion fishbones [8], also electron fishbone nonlinear dynamics is expected to be dominated by wave–particle nonlinear interactions, as originally proposed by Chen et al [9], and confirmed by numerical simulations in [16]. The role of MHD nonlinearity can also be important [26, 27], sufficiently near marginal stability. The main aim of this work, however, is to clearly identify the physics processes underlying nonlinear mode dynamics and energetic electron transports well above the fishbone excitation threshold [9]. Thus, we neglect MHD nonlinearity but fully account for nonlinear wave–particle interactions nonperturbatively [28, 29], using hybrid MHD-gyrokinetic simulations [30]. The general features of simulation results, as far as downward mode frequency chirping accompanying particle transport by secular outward motion, followed by mode saturation, are the same as those discussed by Fu et al for the ion fishbone [16]. The original and novel issues addressed and explained in the present investigation are the physics processes underlying frequency chirping and mode saturation by resonant particle pumping out of the radial region where the wave–particle power is initially localized, which is shown to be consistent with the mode particle pumping mechanism, originally introduced by [23] for explaining secular fast-ion losses due to fishbones in Poloidal Divertor Experiment (PDX) [8]. Note, however, that the numerical approach presented in this paper can be extended to the investigation of regimes in which significant asymmetries between electron and ion fishbones arise.

In the following, the numerical code and setup of the ‘numerical experiment’ will be presented first (section 2). Then, the linear dynamics will be described along with the relation between unstable mode and single particle characteristic resonances (section 3). In section 4, the nonlinear saturation and the energetic electron transport related to e-fishbone instabilities will be discussed. The use of energetic particle phase-space diagnostics [31], based on the Hamiltonian mapping techniques generating kinetic Poincaré plots [32], will allow us to clearly identify the physics process underlying fishbone mode saturation, frequency chirping and secular (versus diffusive) energetic particle redistribution. Finally, concluding remarks and discussions are given in section 5.

2. Numerical simulations

The hybrid MHD-gyrokinetic model [30] has been proven very successful in describing the interaction between Alfvén waves and energetic particles in toroidal devices. The HMGC code [28], based on the hybrid MHD-gyrokinetic model [30] and originally developed at the Frascati laboratories, has been applied to studies of energetic particle driven modes (such as toroidal Alfvén eigenmodes (TAEs) and energetic-particle modes (EPMs) [29, 33, 34]), as well as to the analysis of modes observed in existing devices (JT-60U [35], DIII-D [36]) or expected in forthcoming burning plasmas (ITER [37, 38]) and proposed experiments (FAST [39–41]). The simple physical model, originally used in HMGC (O(ε)) nonlinear reduced MHD equations, circular shifted magnetic surface equilibrium, zero bulk plasma pressure and drift-kinetic fast ions) has been recently extended to include new physics [20]. These extensions (from which the name XHMGC) include kinetic thermal ion compressibility and diamagnetic effects, in order to allow for an entirely novel treatment of the realistic (enhanced) inertia response [5, 24, 25] and ion Landau damping [10]. Moreover, XHMGC is now able to evolve up to three independent particle populations kinetically using the particle-in-cell numerical scheme, assuming different equilibrium distribution functions (as, e.g. bulk ions, energetic
ions and/or electrons accelerated by neutral beam injection (NBI), ICRH, fusion generated alpha particles, etc.

The FTU-like equilibrium (see, e.g. [5]) that will be used hereafter, corresponds to a torus with circular shape cross-section, with an inverse aspect ratio \( \epsilon \equiv a/R_0 \approx 0.3 \) (with \( a \) and \( R_0 \) the minor and major radius, respectively). The safety factor \( q \) profile is assumed corresponding to a slightly reversed magnetic shear, with \( q_0 \approx 1.25, q_{\text{min}} \approx 1.05 \) at \( r_{\text{min}}/a \approx 0.35 \) and \( q_a \approx 6 \). Reference on-axis magnetic field \( B_T = 5.4 \text{T} \), deuterium bulk plasma with on-axis density \( n_0 = 1 \times 10^{20} \text{m}^{-3} \) and profile \( n_i(\psi)/n_0 = (1 - \psi)^{1/2} \), on-axis ion temperature \( T_0 = 2 \text{keV} \) and radial profile \( T_i(\psi)/T_0 = (1 - \psi) \) are assumed as well. Here \( \psi \) is the normalized poloidal flux, ranging from \( \psi = 0 \) on axis and \( \psi = 1 \) at the plasma boundary. The motivation of our present choices for the core plasma profiles is that they represent both those used in the record FTU discharges [21, 22], achieving \( T_0 \lesssim 15 \text{keV} \) with line density \( \approx 4 \times 10^{19} \text{m}^{-3} \) in experiments with ECRH in the plasma current ramp phase; as well as those of electron fishbone observation in FTU LH discharge [5]. They, thus, bear the necessary ingredients for future applications to actual foreseen experiments in FTU, with on-axis ECRH and/or combined central ECRH/LHCD [21, 22], as noted in section 1.

As to energetic electrons, simulation particles are loaded according to the following \( f_{\text{electrons}} \) distribution function:

\[
f_{\text{electrons}} \propto \frac{n_{\text{Ee}}(\psi)}{T_e(\psi)^{3/2}} \Xi(\alpha; \alpha_0, \Delta) e^{-E/T_e(\psi)} \]

\[
\Xi(\alpha; \alpha_0, \Delta) \equiv \frac{4}{\Delta \sqrt{\pi}} \exp \left[ -\left( \frac{\cos \alpha - \cos \alpha_0}{\Delta} \right)^2 \right] \text{erf} \left( \frac{1 - \cos \alpha}{\Delta \sqrt{2}} \right) + \text{erf} \left( \frac{1 + \cos \alpha}{\Delta \sqrt{2}} \right),
\]

where \( E \) is the single particle energy, \( n_{\text{Ee}}(\psi) = n_{\text{Ee}0} \exp(-10\psi) \) is the radial density profile, \( T_e(\psi) \) is the radial temperature profile, \( v_i \) is the parallel (to the equilibrium magnetic field) velocity, \( \alpha \) is the conserved magnetic moment, \( \alpha \) is the pitch angle of the energetic electrons, \( \Xi(\alpha; \alpha_0, \Delta) \) models the anisotropy of the distribution function and \( \Omega_{\text{ce}} = eB/(m_e c) \) is the cyclotron frequency, with \( e \) and \( m_e \) being the (absolute value of) charge and mass of electrons, respectively, and \( B \) the local equilibrium magnetic field. The parallel velocity is normalized to the on-axis energetic electron thermal velocity \( \hat{u} \equiv v_{i0}/v_{\text{th},e,0} \) with \( v_{\text{th},e,0} = \sqrt{T_{\text{e0}}/m_e} \) and the magnetic moment is normalized as \( \hat{\mu} \equiv \mu \Omega_{\text{ce}}/T_{\text{E0}} \), with the subscript '0' indicating on-axis values. A distribution function characterized by perpendicular temperature much higher than the parallel one is considered in the simulations by assuming \( \cos \alpha_0 = 0 \) and \( \Delta = 0.1 \). The anisotropic part of the electron distribution function \( f_{\text{electrons}} \) in the normalized space \( (\hat{\mu}, \hat{u}) \) and the \( \psi \)-dependence of the energetic electrons density are shown in figure 1 (the localization of the maximum gradient of the energetic electrons density is somewhat internal but close to the minimum \( q \) position). Energetic electrons are assumed to be characterized by uniform temperature \( T_{\text{E0}} = T_{\text{E0}} = 50 \text{keV} \); thus, \( v_{\text{th},e,0}/v_{\text{A0}} \approx 11.27 \) (with \( v_{\text{A0}} \) being the on-axis Alfvén velocity), while the on-axis energetic electrons Larmor radius is \( \rho_{\text{Ee}}/a \approx 3.5 \times 10^{-4} \). As noted in section 1, the choice of an on-axis peaked supra-thermal electron distribution function allows us to focus on the simplified proof-of-principle case, where e-fishbones rotating in the electron diamagnetic direction are excited by trapped electrons. Meanwhile, the phase-space anisotropic distribution focuses the dynamics of deeply trapped particles only, yielding the aforementioned analogies with ion fishbone nonlinear behaviours, which give a character of generality to the results obtained in this work.

We observe that the supra-thermal electron distribution function, as previously assigned, is given in terms of variables \((E, \alpha, \psi)\). While \( E \) is a constant of the unperturbed motion, \( \psi \) and \( \alpha \) are not (though \( \psi \) is approximately constant because of the small electron orbit width). We can then expect that the distribution function relaxes in time. To prevent such relaxation, in the reported simulations, the \( \nabla_B \) drift
Energetic electron driven mode: total energy content in the different Fourier components used in the simulation (a). No unstable mode is observed (b) in a similar simulation in which, artificially, the contribution of trapped energetic electrons to the driving term is switched off. On-axis energetic electron density normalized to bulk ion density is \( n_{Ee0}/n_B = 0.2 \).

Figure 2. Energetic electron driven mode: total energy content in the different Fourier components used in the simulation (a). No unstable mode is observed (b) in a similar simulation in which, artificially, the contribution of trapped energetic electrons to the driving term is switched off. On-axis energetic electron density normalized to bulk ion density is \( n_{Ee0}/n_B = 0.2 \).

Contribution to the source term in the equation that evolves the weight of the particles is neglected, thus forcing the initial distribution function to behave as an equilibrium one. In order to properly account for thermal ion Landau damping, enhanced plasma inertia (mostly due to trapped thermal ions) and finite compressibility, also the bulk ions are treated kinetically using the particle-in-cell numerical scheme, assuming that their initial distribution function is an isotropic Maxwellian with \( v_{i0}/v_{A0} \approx 3.72 \times 10^{-2}, \rho_i/a \approx 4.27 \times 10^{-3} \). The contribution from the divergence of thermal ions pressure tensor, as computed by the gyrokinetic module of XHMGC, is added to the generalized vorticity equation on the same footing of the energetic particle pressure tensor \([30]\) (for details on the model and numerical implementation see equation (11) and appendix A of \([20]\)). In such a way, the thermal ion Landau damping as well as the generalized inertia, with the actual dynamic response of trapped and circulating thermal ions, are consistently retained (see also section 2.2 and appendix A of \([5]\)). Note that ion Landau damping and enhanced plasma inertia generally play an important role for both ion and electron fishbones physics \([5, 10, 24, 25]\). However, while the role of (linear) ion Landau damping is obvious, as it impacts the threshold condition for mode excitation, the effect of enhanced plasma inertia is more subtle. It is shown that enhanced plasma inertia crucially depends on magnetically trapped particles and that, in the low-frequency limit, it reduces \([5, 10]\) to the neoclassical zonal flow polarizability \([42]\). This impacts the relative role of energetic particle drive and continuum damping, as the latter is proportional to plasma inertia enhancement; thus, it implies a higher energetic particle threshold condition for precessional fishbone excitation \([7]\). As to nonlinear physics, neoclassical zonal flow polarizability occurs in the criterion for dominant energetic particle nonlinearity over zonal flows and fields \([43]\). More recently, it has also been shown that enhanced plasma inertia may play a significant role in determining the relative role of zonal flows and fields in saturation of Alfvenic and MHD fluctuations \([44]\).

Here, it is worthwhile reminding that, as already noted in section 1, we neglect MHD nonlinearity in our numerical simulations, but fully account for nonlinear wave–particle interactions nonpertubatively \([28, 29]\).

Considering particle dynamics, it has to be noted that energetic electrons are characterized by much larger velocities than bulk ions. Thus, a time step sub-cycling algorithm is implemented, such that, at a fixed field-solver time step, each numerical particle chooses its own time step sub-division to properly integrate the equations of motion in the gyrokinetic module. This algorithm is particularly suited to follow carefully the simulation particles near the origin of the (polar) coordinate systems, which, as usual, a delicate point in configuration space to be treated numerically.

3. Linear dynamics

A simulation showing an unstable mode driven by the energetic electrons is reported in figure 2(a): the toroidal mode number is \( n = 1 \), the poloidal Fourier components retained are \( m = 1, \ldots, 6 \) and the on-axis energetic electron density normalized to bulk ion density is \( n_{Ee0}/n_B = 0.2 \). The mode exists only if a threshold in \( n_{Ee0}/n_B \) is exceeded, which, for this particular equilibrium, is \( n_{Ee0}/n_B \approx 0.108 \) (see figure 3, where the growth rate and real frequency are shown versus the energetic electron density).

A very clear growing mode, with the characteristics of an \( n = 1 \) internal kink is observed, with a somewhat different displacement function from the classic \( m = 1 \) ‘step function’, because of the weakly reversed shear inside the \( q_{min} \) surface used in the simulation. This can be seen in figure 4, where the poloidal structure \((a)\), the power spectrum of the scalar potential \((b)\) and of the parallel vector potential \((c)\) of the fluctuating electromagnetic field are shown for the case with \( n_{Ee0}/n_B = 0.13 \). Note that the mode rotates counterclockwise, which, for the considered equilibrium, corresponds to the direction of supra-thermal electron diamagnetic velocity. A first evidence that the mode shown in figure 2(a) is actually...
an e-fishbone, can be obtained by artificially switching off the contribution of trapped supra-thermal electrons in the driving term, showing that the system, in this case, is stable; see figure 2(b).

To better clarify the dynamics of energetic particles, the power transfer from the energetic electrons to the wave during the linear growth of the mode is shown in figure 5(a) versus \( \mu \) and \( \bar{u} \) for the same case as in figure 4, at the radial position where the power exchange is maximum \((0.14 \lesssim r/a \lesssim 0.19)\). Figure 5(b) shows the analogous quantity for thermal ions (with \( \mu \) and \( \bar{u} \) normalized, in this case, in terms of ion quantities).

Trapped energetic electrons yield a net drive to the wave, whereas Landau damping is given mainly by counter-passing bulk ions, and, in minor part, by co-passing ones. In figure 5, the trapped/circulating particle boundary for the radial region considered, as obtained in the large aspect ratio limit are also reported, as reference. Note that, in figure 5, each particle contribution is referred to the value of \( \bar{u} \) of that particle when it crosses the equatorial plane at poloidal angle \( \theta = 0 \). The contribution of counter-passing and co-passing fractions is much lower, in amplitude, than that of magnetically trapped fast electrons, is localized close to the trapped/circulating boundary and causes little damping. This is consistent with the fact that barely trapped/circulating electrons are characterized by precession reversal [5].

While performing a numerical simulation, the XHMGC code can also be used to follow a set of test particles; the phase-space coordinates of such particles are stored in time, and can be used to compute a variety of single particle physical quantities. In this case, we are interested in analysing the single particle frequencies of the supra-thermal electrons, namely, the precession and bounce frequencies. To this purpose, a set of test particles is initialized at the radial location where the power exchange is maximized, and covering uniformly the \((\mu, \bar{u})\) region of phase space shown in figure 5(a). The bounce frequencies of energetic electrons are always much larger, in absolute value, than the mode frequency, whereas the precession frequencies of the trapped test electrons fall in the range of the mode frequency. In figure 6, the power exchange between trapped energetic electrons and the wave (see also figure 5(a)) is shown along with the curve describing the resonance condition \( \omega_0(\mu, \bar{u}) - \omega = 0 \), with \( \omega \) obtained by the simulation \((\omega = \omega_{0,13} \approx -0.095\omega_{A0}, \text{where } 0.013 = \omega_i/\omega_D/\mu = 0.13\), see also figure 3), and \( \omega_0 \) the precession frequency of the trapped electrons determined numerically from test particle motion. Also, the two curves corresponding to \( \omega_0(\mu, \bar{u}) - (\omega_{0,13} \pm 3\gamma_{0,13}) = 0 \), with \( \gamma_{0,13}/\omega_{A0} \approx 0.02 \), are shown in order to give a qualitative feeling of the natural spread in real frequency induced by the finite mode growth rate. As can be seen from figure 6, the regions in \((\mu, \bar{u})\) space where the maximum power exchange occurs (red colour in the figure), compare well with the curve which corresponds to the mode frequency observed in the simulation (black, thick curve).

4. Nonlinear dynamics

Taking as reference the weakest growth rate \((n_{e0}/n_A = 0.12)\) case shown in figure 3, the time evolution of the simulation can be described as follows: during the linear growth-rate phase (figure 7(b)), the mode driven by trapped energetic electrons builds up in the region inside the minimum \( q \) surface \((r/a < 0.35)\), with real frequency \( |\omega| \) above the minimum tip of the lower Alfvén continuum. When the nonlinear regime is entered (figure 7(c)), \( |\omega| \) chirps towards the lower tip of the Alfvén continuum, while further in the nonlinear phase (figure 7(d)), other oscillations, in correspondence of subsequent bursts in the stored energy of the poloidal Fourier components, are eventually observed at larger \( |\omega| \).

Figure 8 shows the energetic electron density for \( t/\omega_{A0} = 550 \) (a), \( t/\omega_{A0} = 900 \) (b) and \( t/\omega_{A0} = 1248 \) (c). At each time step, the radial profile of the \( m = 0, n = 0 \) particle density component is plotted. Contour plots in the poloidal plane are also reported separately, both for circulating (centre) as well as trapped (right) particle contributions. Macroscopic modification of the energetic electron density profile is clearly observed. In particular, a steepening of the radial profile, inside and outside the original region of maximum radial gradient is evident in the late nonlinear phase and it is a plausible cause of the secondary bursts observed. Moreover, the 2D plots of circulating and trapped energetic electron densities show that significant velocity space transport is taking place (as a counterpart of the analogous process in real space), due to nonlinear mode dynamics. Note that XHMGC adopts a coordinate system with the radial coordinate starting from the geometrical axis, not coincident with the magnetic axis, where the initial energetic electron density is peaked; this is why the \( m = 0, n = 0 \) harmonic of such profile is not peaked at \( r = 0 \).

The behaviour of resonant energetic electrons during the earlier nonlinear phase \((t/\omega_{A0} < 900, \text{see figure 7(a)})\) can be investigated, following [32], by selecting a set of test particles characterized by certain values of the two constants of the (perturbed) motion, \( \mu \) and \( C = \omega P_e - nE \), where \( P_e = m_i R_v |t| - e\psi/c \) is the toroidal canonical angular momentum.\(^3\)

\(^3\) Note that this form of the C constant is the lowest order expression for the conserved Hamiltonian in the extended phase space (see, e.g. [45]), obtained by asymptotic expansion in \(|\omega/\omega^2| \ll |\gamma/|\omega| \ll 1).
Figure 4. Poloidal structure of the energetic electron driven mode (electrostatic component of the fluctuating electromagnetic field, (a)), its power spectrum (b) and power spectrum of the fluctuating parallel component of the vector potential (c); the black curves represent the Alfvén continuous spectrum.

Figure 5. Power exchange in the $(\hat{\mu}, \hat{u})$ space of, respectively, supra-thermal electrons (a) and the bulk ions (b) with the wave at the radial position where the electron power exchange is maximum ($0.14 \lesssim r/a \lesssim 0.19$) (violet colour code corresponds to maximum damping, red to maximum drive). The black curves correspond to the trapped/circulating particle boundary at $(r/a)_{\text{min}} = 0.14$ (solid line), and at $(r/a)_{\text{max}} = 0.19$ (dashed line). Note that velocity space variables, normalizations and scales of the electrons and ions are different as discussed in the text.

Figure 6. Power exchange, in the $(\hat{\mu}, \hat{u})$ space, between the trapped energetic electrons and the wave, with superposed curves $\omega_0 - \omega = 0$ for three values of $\omega$: $\omega_{(0,1)}/\omega_A \approx -0.095$ (black thick curve), $(\omega_{(0,1)} + 3\gamma_{(0,1)})/\omega_A \approx -0.035$ (red curve) and $(\omega_{(0,1)} - 3\gamma_{(0,1)})/\omega_A \approx -0.155$ (blue curve). Here $\omega_0$ is obtained from the motion of a set of test particles.

The specific values of $\mu$ and $C$ are chosen in such a way that they correspond, during the linear phase, to a resonant region in the phase space with strong power transfer from particles to waves (see, e.g. the red region in figure 5(a)). Different test particles are then loaded with different radial coordinates within a shell around the mode peak and corresponding different values of $v_0$. Whenever a test particle completes a full banana or transit orbit in the poloidal plane at $t = \hat{t}$, by crossing the outer equatorial plane ($\theta = 0$), the corresponding values of the toroidal canonical momentum $P_\phi(\hat{t})$ and the wave–particle phase $\Theta_1(\hat{t})$ are computed, with $\Theta_1(\hat{t}) = \int_0^{\hat{t}} \omega(t) \, dt - n\phi(\hat{t})$ and $\phi$ being the toroidal angle. Each test particle position in the plane $(\Theta_1, P_\phi)$ is then updated by moving the marker in a ‘kinetic Poincaré’ plot [32]. In figure 9 a series of frames is reported, showing kinetic Poincaré plots at different times; we have considered the resonant region around $\hat{\mu} \approx 3$, $\hat{u} \approx -0.3$. The $P_\phi$ initialization of test particles ($-0.13 \lesssim \hat{P}_\phi \equiv P_\phi/m_e v_{\text{th},e} \lesssim 76.2$) corresponds to radial coordinates $0.1 \lesssim r/a \lesssim 0.35$. The colour of each marker depends on the initial $P_\phi$ value of the corresponding test particle, in order to make following marker motion in the $(\Theta_1, P_\phi)$ plane easier. $P_\phi$ is an invariant of the unperturbed motion. For a certain value of $P_\phi (P_{\phi,\text{res}})$, particles will be exactly in phase with
Figure 7. Time evolution of the total (kinetic plus magnetic) energy stored in the different poloidal Fourier components in log scale (a) and power spectrum of the electrostatic field at three different times: $t_{\omega A_0} = 300$ ((b), linear phase of the simulation), $t_{\omega A_0} = 900$ ((c), early saturated phase) and $t_{\omega A_0} = 1248$ ((d), late saturated phase).

The wave (constant $\Theta$). Such resonant particles will then be represented, in the linear phase of the simulation (negligible perturbed field amplitude; figure 9(a)), by approximately fixed markers. Particles with different values of $P_\phi$, instead, will exhibit a drift in $\Theta$. In this simulation, the resonance condition corresponds to $P_{\phi,\text{res}} \approx 10$, and we find $\Theta \gtrsim 0$ for $P_\phi \lesssim P_{\phi,\text{res}}$, respectively. As the effect of the perturbed field becomes relevant, $P_\phi$ is no longer conserved, and particles drift also in the $P_\phi$ direction (e.g. because of the $\psi$ variation due to $E \times B$ drift). The sign of $P_\phi$ variation will depend on the value of the wave–particle phase $\Theta$ between (nearly) resonant particles and the perturbed field. Therefore, particles sufficiently out of resonance with the wave (i.e. far from the resonant $P_\phi$) are just modulated in $P_\phi$, while fast drifting in $\Theta$. Particles originally close to the resonance condition have instead the tendency to move on closed trajectories (‘trapped’), so that it is possible to identify a separatrix between these two behaviours in the instantaneous potential well of the wave. In the second frame ($t_{\omega A_0} = 550$, figure 9(b)), this can be seen for particles with $3 \lesssim P_\phi \lesssim 16$. As the mode amplitude grows, the $P_\phi$ width of the instantaneous separatrix increases, as shown in figure 9(c), corresponding to $t_{\omega A_0} = 750$. Meanwhile, the separatrix structure drifts significantly upwards in the $(\Theta, P_\phi)$ plane in the third (figure 9(c)) as well as in the fourth frame ($t_{\omega A_0} = 900$, figure 9(d)).

In figure 10, the trajectories of two resonant particles are reported in the $(\Theta, P_\phi)$ plane. Colours of each dot are related, in these plots, to the instantaneous wave–particle power exchange: red corresponds to the maximum destabilizing particle contribution; violet, to the maximum stabilizing effect.

First, we note that the bounce time of these particles $\tau_B$, i.e. the inverse characteristic rate $|\dot{\Theta}|^{-1}$, is of the same order of the mode saturation time $\tau_{NL}$ (see figure 7(a)). Second, the power transfer from particles to wave along the trajectory does not average to zero and is connected with rapid frequency chirping, $\dot{\omega} \approx \tau_B^{-2}$, consistent with secular motion of a significant fraction of resonant particles and the ‘mode particle pumping’ process [23]. This implies that the saturation of the mode cannot be ascribed to a fast averaging of the stabilizing and destabilizing contributions of particles that are trapped in the wave. This interpretation is supported by the results shown in figure 11(a), where evolution of the precession frequency of the linearly resonant particles is reported. More specifically, defining, for the $i$th particle, the precession frequency as $\omega_{D,i} \equiv n d\phi_i(t)/dt$, we represent the distribution of such quantity over the linearly resonant particles by plotting its average $\langle \omega_{D} \rangle \equiv \sum_i p_{lin}^{i} \omega_{D,i}(t)/\sum_i p_{lin}^{i}$ (solid red line) and spread $\delta \omega_D \equiv \sum_i p_{lin}^{i} [\omega_{D,i}(t) - \langle \omega_{D} \rangle]^{2}/\sum_i p_{lin}^{i}\delta\omega_D^{(i)}$ (dashed red lines). Here, $\sum_i$ is extended over all the linearly driving test particles, and $p_{lin}^{i}$ is the average power transferred from the $i$th particle to the wave during the linear phase. We see that the absolute value of $\omega_{D}$ decreases with time, as soon as particle motion is perturbed by the fluctuating field, as a result of $\omega_{D}$ dependence on the
Figure 8. Energetic electron density radial profile (left), and 2D plots in the poloidal plane of circulating (centre) and trapped (right) populations at three different times: \( t\omega_A = 550 \) (linear phase of the simulation, (a)), \( t\omega_A = 900 \) (early saturated phase, (b)) and \( t\omega_A = 1248 \) (late saturated phase, (c)).

... particle radial position and the net outward radial particle displacement. The mode, meanwhile, reacts by adjusting its frequency, in order to minimize resonance detuning, i.e. \(|\dot{\Theta}|\), and maintain the ‘phase-locking’ condition, consistently with the mode particle pumping mechanism \([5, 23, 46]\). This effect can also be appreciated directly in figure 11(b), where the curves refer to the quantity \( d\Theta(t)/dt \), with average and spread values defined as for \( \omega_p \). Note that phase-locking is the reason for frequency chirping (see also figures 7(b) and (c)) and for the observed upward drift in \( P_\phi \), with almost constant \( \Theta \), exhibited by the resonant particles in figure 10. In the absence of phase-locking, particles detune faster w.r.t. the mode (with consequent loss of drive), as can be seen from the red curve in figure 11(b), corresponding to the average value of \( d\Theta(t)/dt \) for constant mode frequency. In spite of phase-locking, the mode eventually cannot preserve the initial drive, because the radial displacement of linearly resonant particles causes them to experience a reduced mode amplitude, due to radial mode structure and radial nonuniformities. This can be seen from figure 12, comparing \( \bar{\mathcal{P}}(t) \) and \( \mathcal{P}(t) \pm \delta r(t) \) with the linear-phase radial structure of the poloidal electric field. Furthermore, phase-locking is not exact, as demonstrated by the resonance broadening shown in figure 11(a). The combined effect of all these factors produces the saturation of the linearly dominant mode. This saturation is then followed, as described above, by a more complicated, fully nonlinear dynamics, which is not the object of our present investigation.

5. Discussions and conclusions

In this paper, the results of the first nonlinear numerical simulation of supra-thermal electron driven internal kink instabilities are presented. These instabilities have been observed in several experiments in tokamaks in the presence of electron cyclotron heating and/or lower hybrid heating and current drive. They have a specific relevance in the frame of the investigation of the expected burning plasma properties, as high-energy electrons are characterized by a
Figure 9. Kinetic Poincaré plots at four different times (see figure 7(a)): $t_{ωA0} = 300$ (a) and $t_{ωA0} = 550$ (b) (early and late linear phase of the simulation), $t_{ωA0} = 750$ (c) and $t_{ωA0} = 900$ (d) (saturation phase); test particles are coloured corresponding to their birth value of $P_φ$. Note that the extension of the $Θ$-axis is doubled and test particle markers are replicated in the domain ($2π < Θ < 4π$) to enhance the readability of the plots.

Figure 10. Trajectories of two resonant particles in the kinetic Poincaré. Different times are indicated along each trajectory.

very small ratio between the resonant particle orbit width and the extension of the characteristic fishbone length scale. This feature is expected to characterize ion fishbones in burning plasmas of fusion interest due to the large plasma current, while it is not realized for the energetic ions in present-day experiments. Meanwhile, fluctuation induced transport of magnetically trapped resonant particles, due to precession resonance, is expected to depend on energy and not mass because of the bounce averaged dynamic response. Then, studying the coupling between MHD modes and energetic
**Figure 11.** Comparison between $\omega_D(t)$ (solid red line) and $\omega(t)$ (black line) for the linearly resonant particles (a). The width of the $\omega_D$ distribution is also reported by plotting the quantities $\omega_D \pm \delta\omega_D$ (dashed red lines). Average (solid black line) and distribution width (dashed lines) for the quantity $d\Theta_1(t)/dt$ (b). The average value of $d\Theta_1(t)/dt$ that would be obtained for constant mode frequency is also plotted for comparison (b, red line).

**Figure 12.** Left: radial profile of the harmonics of the poloidal electric field in the linear phase (in red colour the $m=1$ component; note that the vertical axis is the radial position up to $r/a=5$). Right: average radial position of the linearly resonant particles (black line); the width of the radial position distribution of such particles is also indicated by the red curves.

electrons allows new insights into the interaction between alpha particles and MHD modes in next generation tokamaks.

A numerical experiment is carried out by the hybrid MHD-gyrokinetic code XHMGC, describing the system in terms of reduced MHD equations coupled with kinetic contributions that are needed for the description of finite plasma compressibility and resonant wave–particle interactions involving thermal ions and energetic electrons. We have addressed electron fishbone excitation by a population of magnetically trapped supra-thermal electrons with pressure profile peaked on axis, generated, e.g. by electron cyclotron resonance heating. With the aim of identifying the relevant physical processes determining the nonlinear dynamics of the system well above the threshold for the destabilization of the mode, MHD nonlinearities are neglected, while nonlinear wave–particle interactions are fully retained. The mode is excited by precession resonance with magnetically trapped electrons. Counter-passing thermal ions tend, instead, to stabilize the mode via Landau damping.

The nonlinear dynamics of the mode is investigated through Hamiltonian mapping techniques. To this aim, a set of resonant test particles is evolved in the fluctuating field computed by the self-consistent simulation, and their phase-space coordinates are stored at each time the particle orbit crossed the outboard equatorial plane. The corresponding trajectories in the $(\Theta, P_\phi)$ phase space have then been mapped. From this analysis, using a sufficiently high number of test particles, it is possible to visualize the instantaneous separatrix between particles that follow trapped versus open trajectories in the phase space. As mode structure and/or frequency evolve in time, the instantaneous separatrix is consistently modified. Thus, for nonadiabatic nonlinear wave–particle dynamics, the instantaneous separatrix does not provide information on actual phase-space trajectories of physical particles, but it gives nonetheless useful insights into the changing wave–particle resonance structures in phase space. This is the case of the nonlinear electron fishbone dynamics, investigated in this work, for which the characteristic particle bounce time within the instantaneous separatrix (assuming that a closed orbit is actually completed) is of the same order of the mode saturation time. Thus, particles are not effectively trapped by the wave and mode saturation cannot be traced back to a fast averaging of destabilizing and stabilizing contributions of these particles along their periodic motion, as it would be for the case of a beam–plasma system [47]. A careful analysis of resonant particle dynamics shows that saturation is rather due to the fact that energetic electron precession frequency decreases in time, because of their secular outward displacement caused by the interaction with the fluctuating field. In order to maximize wave–particle power exchange with resonant particles, the mode minimizes resonance detuning...
(the rate of change of the wave–particle phase) adjusting its frequency, giving rise to a downward frequency chirping, consistently with the experimental observations of electron fishbones. This process is feasible, due to the intrinsic nature of the fishbone mode to be born out of the shear Alfvén continuum spectrum at the characteristic frequency of supra-thermal particles, corresponding to the fastest growing mode [9]. In fact, the readiness of wave-packets of the Alfvén continuum to respond at the optimal frequency ensuring maximum resonant wave–particle power exchange is what allows the resonance condition to be maintained by phase-locking, with the corresponding outward secular motion and transport of energetic particles, till mode saturation occurs due to radial particle decoupling from the mode structure. The frequency dependence of continuum damping also plays an important role in the nonlinear fishbone dynamic evolution; e.g. continuum damping typically decreases for decreasing mode frequency, thereby facilitating the preservation of phase-locking and secular outward mode particle pumping for a longer time.

In summary, numerical simulation results demonstrate that nonlinear fishbone dynamics is governed by frequency chirping due to phase-locking, accompanied by mode particle pumping, originally introduced by White et al [23]. Meanwhile, the ejection of resonant particles from the region where the wave–particle power exchange is initially localized, consistent with previous results discussed by [16] and analysed here by Hamiltonian mapping techniques, produces radial decoupling of resonant particles from the mode structure. Numerical simulations also show that phase-locking is not exact, and that there exists an intrinsic residual resonance detuning, causing resonance broadening, connected with radial nonuniformities of characteristic energetic particle resonant frequencies. Mutatis mutandis, this effect is analogous to that discussed by Shapiro [48], who pointed out that, in the nonlinear interaction of a plasma with a monochromatic beam, the beam is overall slowed down, while exciting the underlying beam–plasma instability, but is also heated (broadened) by the nonlinear beam–plasma interaction. Eventually, these processes yield mode saturation.

As specified in section 1, this work is aimed at analysing in detail the dominant nonlinear fishbone dynamics well above excitation threshold, providing new insight into the underlying physics processes. This situation is not only of theoretical interest but could also be of practical relevance to describe future experiments characterized by an anisotropic supra-thermal electron distribution function peaked on axis, produced by strong on-axis ECRH and/or combined central ECHR/LHCD. Much richer physics is expected to become increasingly more relevant as plasma conditions closer to marginal stability are approached; e.g. MHD nonlinearities cannot be neglected. Correspondingly, more theory and simulation studies are needed to fully understand and explain the diverse experimental evidence, recently reported and summarized by [14].

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