Particle Simulations of Alfvén Modes in Reversed-Shear DIII-D Discharges Heated by Neutral Beams*

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Abstract. A rich spectrum of oscillations in the Alfvénic range has been observed in DIII-D tokamak reversed-shear discharges heated by neutral beams. During the discharge phase characterized by Alfvénic activity, the energetic particle density profile, as calculated by a classical deposition model (TRANSP), appears to be much more peaked than that observed experimentally, as inferred, e.g., from the equilibrium reconstruction. In this paper the results of particle simulations of Alfvénic modes driven unstable by energetic ions in reversed-shear DIII-D discharges, performed by the HMGC code, are presented. Single toroidal mode number simulations ($n = 2 \div 4$), which fully retain energetic particle nonlinearities, are considered. An investigation of the sensitivity of the simulated-mode frequency to the q-profile variations is presented, along with a comparison with the experimental results. On the basis of our simulations the following interpretation of the DIII-D Alfvénic/energetic-particle phenomenology can be drawn: a) if the Alfvén modes were ineffective, the energetic ion density profile would be close to that obtained by TRANSP simulations; b) with Alfvén mode dynamics included in the simulations, the TRANSP fast ion profile would generate strongly driven modes (EPM, with frequencies mainly determined by the energetic ions), inducing significant transport and flattening such profile on a time scale $\tau \leq 100\mu s$; c) as the drive is reduced by the flattening of the energetic ion density profile and/or by the modification of their velocity-space distribution function, residual Alfvén modes exist, close to marginal stability, with frequencies strongly influenced by the q-profile evolution.

1. Introduction

A rich spectrum of oscillations in the Alfvénic range has been observed (see Fig. *1.*, taken from Ref. [1]) in DIII-D tokamak discharges characterized by reversed-shear q-profile and heated by neutral beams [2].

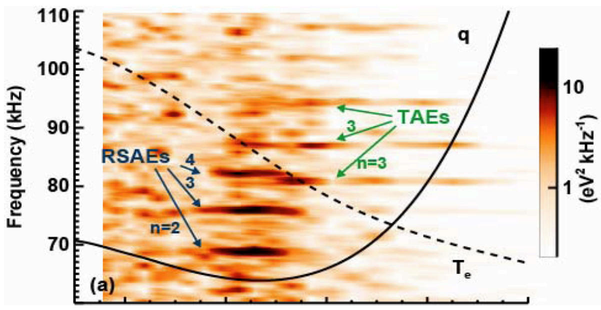

FIG. 1.: Alfvén Eigenmode frequency spectrum observed on DIII-D device, from Ref. [1].

During the phase of the discharge characterized by Alfvénic activity, the energetic particle density profile, as calculated by a classical deposition model (TRANSP code [3]), appears to be much more peaked than the one observed experimentally, as inferred, e.g., from EFIT [4] equilibrium magnetic flux reconstructions that use MSE (Motional Stark effect), magnetic measurements, and thermal pressure data, or from the Fast Ion D_α (FIDA) diagnostic (Fig. *2.*).

In this paper the results of particle simulations of Alfvénic modes driven unstable by ener-

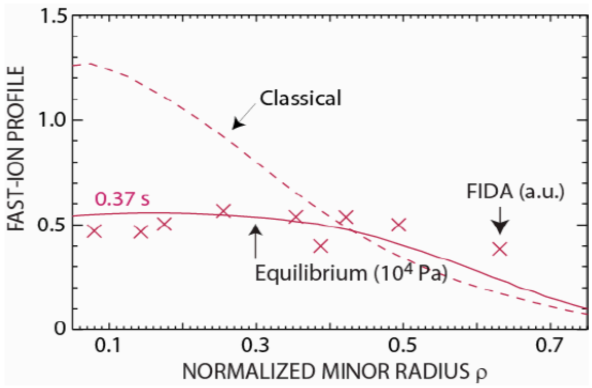

*FIG. 2.: TRANSP (*Classical*) and experimental (*Equilibrium*: kinetic EFIT using MSE and magnetics with subtraction of thermal pressure;* FIDA*: Fast Ion D_α diagnostic) energetic particle profiles on DIII-D, shot number 122117.*

getic ions in reversed-shear DIII-D discharges using the HMGC code [5] are presented. Single toroidal mode number simulations ($n = 2 \div 4$, the dominant modes observed in the experiment), which fully retain energetic particle nonlinearities, are considered. Plasma equilibrium is calculated from q-profile data, assuming shifted circular magnetic flux surfaces. Initial energetic particle distribution function is assumed to have the following form:

$$F = n(r) f_{sd}(E, r) \Theta(\alpha, \alpha_0, \Delta) \, . \tag{1}$$

Here $f_{sd}(E, r)$ is a slowing down distribution function, with birth energy $E_0 = 0.077$ MeV (in the experiment, 2/3 of the beams had birth energy of 0.075 MeV, and the remnant 1/3 had 0.081 MeV) and critical energy, depending on the radial coordinate r, given by the Stix formula [6]. The quantity Θ, given by

$$\Theta(\alpha, \alpha_0, \Delta) \equiv \frac{4}{\Delta\sqrt{\pi}} \frac{\exp\left[-\left(\frac{\cos\alpha - \cos\alpha_0}{\Delta} \right)^2 \right]}{\mathrm{erf}\left(\frac{1-\cos\alpha_0}{\Delta} \right) + \mathrm{erf}\left(\frac{1+\cos\alpha_0}{\Delta} \right)} \, , \tag{2}$$

represents the anisotropy of the distribution function. In this expression, the following definitions have been used: $\cos\alpha \equiv v_\parallel / v$, with $v_\parallel$ being the parallel (to the magnetic field) component of the particle velocity v; $\alpha_0 \equiv \arccos(R_{tan}/R_0) = \arccos(1.15\mathrm{m}/1.688\mathrm{m}) \approx 47\,\mathrm{deg}$ is the injection angle, with R_{tan} and R_0 being the tangential radius of the beam and the major radius of the torus, respectively; Δ is the width of the beam around $\cos\alpha_0$. The value of the parameter Δ can be inferred from the experimental ratio of the parallel to perpendicular energetic particle pressure $p_{par}/p_{perp} \approx 1.44$. Note that this distribution function is not a function of conserved quantities only, and, in fact, it is not an equilibrium distribution function. Indeed, it is observed that the initial distribution function evolves towards an equilibrium one in a quite short time: we conventionally call such an equilibrium the *promptly relaxed* state. This approach is not satisfactory in some cases, as the difference between the initialized energetic particle density profile and the promptly relaxed one is quite relevant. Nonetheless, this representation was chosen since it makes the unique reconstruction of the fast ion distribution function possible from fairly limited experimental data. Meanwhile, this representation is asymptotically correct for large aspect ratio equilibria with small normalized fast ion orbit widths with respect to the machine size.

2. Flattening of the Energetic Particle Density Profile

Assuming the TRANSP profile as initial energetic particle density profile, strongly unstable modes are observed, which cause a violent redistribution of the energetic particles for all the simulated toroidal mode numbers. In Fig. *3*. the energetic particle profiles for the $n = 2$ simulation are shown: the initialized TRANSP profile (black solid curve), the promptly relaxed profile (red long-dashed curve), the saturated profile (blue dashed curve). The saturated profile is strongly modified with respect to the initial (as well as the promptly relaxed) one, in particular in its central value and width. Note that the saturated profile is quite close to the energetic particle profile obtained from equilibrium reconstruction (at time $t = 0.37$ s, see Fig. *2*.), which is also reported for comparison (green dotted curve).

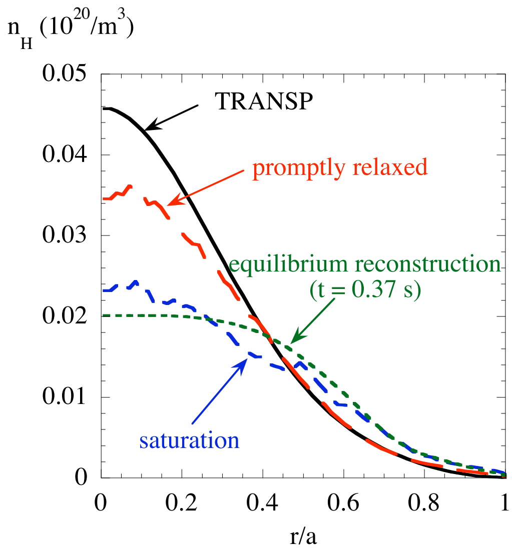

FIG. 3.: Energetic particle density profiles ($n = 2$ simulation): the initialized TRANSP profile (black solid curve), the promptly relaxed profile (red long dashed curve), the saturated profile (blue dashed curve). The energetic particle profile obtained from equilibrium reconstruction at $t = 0.37$ s is also shown (green dotted curve).

In Fig. *4*. the power spectra of the electrostatic potential during the linear growth (top) and the saturated phase (bottom) of the simulation are shown, for the different toroidal mode numbers considered. The dominant modes are localized at the q_{min} location. Several very low frequency modes are observed, in the saturated phase, to be identified as resistive MHD modes (resistivity is necessary for numerics; note that low-frequency activity due to geodesic curvature effects on the shear Alfvén wave is instead not presently included in the HMGC code), along with weaker Alfvén modes localized inside the toroidal gap (at least, in the $n = 4$ case).

From this set of simulations we can draw the following tentative conclusions: (a) the equilibrium profile computed by TRANSP, which neglects energetic particle collective dynamics, is strongly unstable once such dynamics is accounted for; (b) the collective mode dynamics causes a relevant flattening of the energetic particle density profile; (c) the saturated state is close to that reconstructed experimentally (by EFIT and/or FIDA); (d) the experimental profile could then be the byproduct of a short time scale collective phenomenon.

We want to investigate whether the energetic particle distribution that yields the experimentally

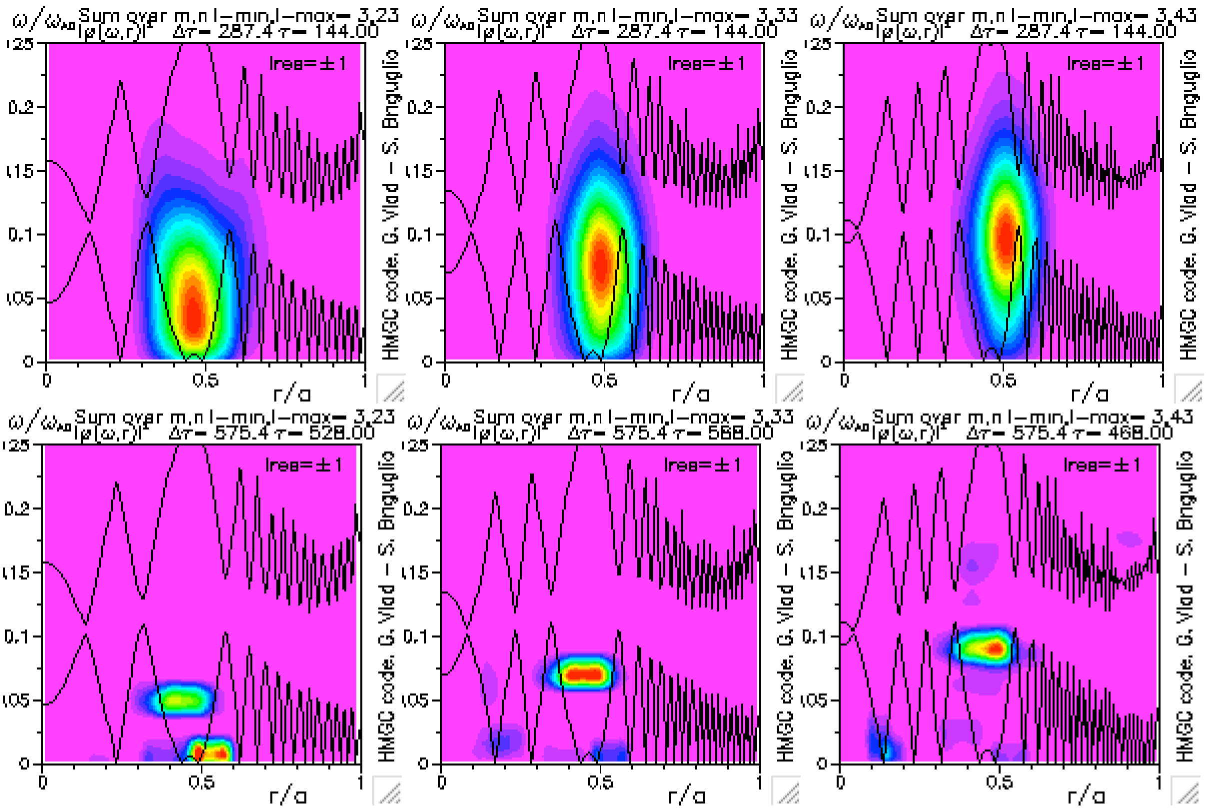

FIG. 4.: Power spectra of the electrostatic potential for $n = 2$ (left), $n = 3$ (center), and $n = 4$ (right). Ordinate is $\omega\tau_{A0}$, the Alfvén time in the center being $\tau_{A0} \approx 2.38194 \times 10^{-7}$ s. Energetic particle initial profile is from TRANSP. Top: linear phase, bottom: saturated phase.

observed density profile (the $t = 0.37$ s profile in Fig. 2.) is distorted from the initial slowing down in the velocity space. If this distortion was small, we would expect that initializing the simulation according to such a slowing-down with the quoted radial density profile should yield almost vanishing instability, with an essentially unaltered saturated density profile. Figures 5. and 6. report the results of the $n = 2$ case. These results show that a sufficiently linearly unstable mode still exists, saturating by a further relaxation of the energetic particle density profile, thus implying that the slowing down distribution function is not a good approximation of the velocity space distribution of the experimentally observed state. This seems to be consistent with the above conclusion (d), taking into account that the saturation of a strongly unstable mode can significantly modify the energetic particle velocity space distribution function, besides their density profile (see Ref. [7] for an analysis of such a phenomenon).

3. Comparison with Experimental Data

Assuming that the experimental fast ion profile results from the effect of a short time scale collective phenomenon, we first check whether the qualitative and quantitative features of the experimentally observed frequency spectra of the modes are well reproduced by HMGC simulations initialized with TRANSP fast ion profile. In this respect, the main qualitative feature is certainly represented by the sensitive dependence of the mode frequency on the minimum-q value, which is typical of the so-called Reversed Shear Alfvén Eigenmodes (RSAEs), also known as Alfvén cascades [8]. Figure 7. (top) shows the power spectra obtained for three different values of q_{min} during the linear mode growth, for $n = 2$. It is apparent that the frequency of the mode is weakly affected by the q_{min} variation and the corresponding modification of the

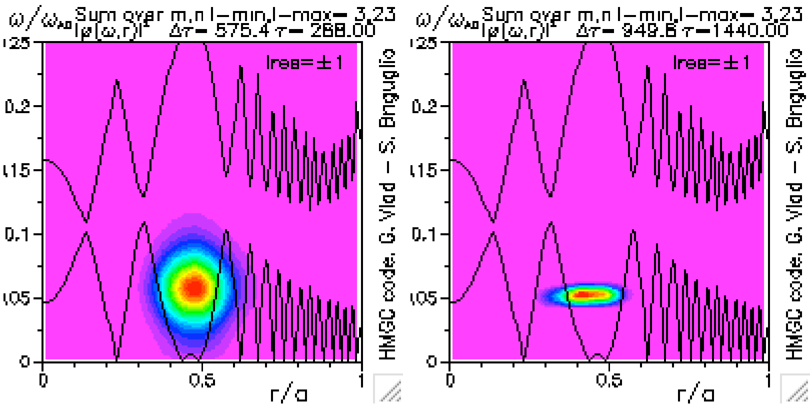

FIG. 5.: Power spectra of the electrostatic potential for a $n = 2$ simulation with energetic particle density profile initialized according to the equilibrium reconstruction at $t = 0.37$ s. The ordinate is $\omega\tau_{A0}$, the Alfvén time in the center being $\tau_{A0} \approx 2.38 \times 10^{-7}$ s. Left: during the linear phase. Right: during the saturated phase.

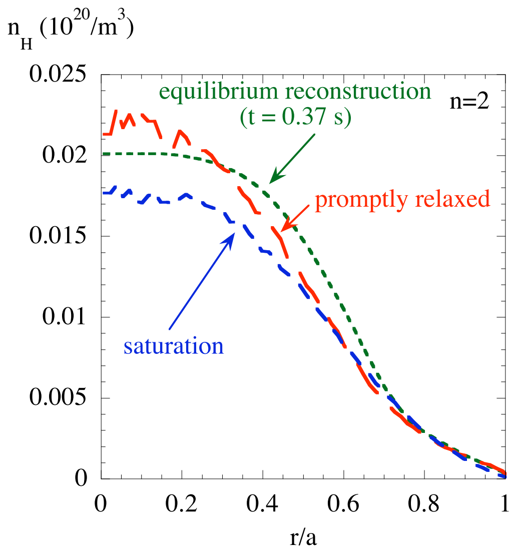

FIG. 6.: Energetic particle density profiles for the simulation considered in Fig. 5.: the initialized profile from equilibrium reconstruction ($t = 0.37$ s, green dotted curve), the promptly relaxed profile (red long dashed curve), the saturated profile (blue dashed curve).

Alfvén continuum. This is not surprising, because the considered modes, in the linear phase, are strongly driven by energetic particles, and hence their frequency is mainly determined by wave-particle resonance condition. We expect that, after the large energetic particle redistribution (saturated phase), the weaker residual modes recover the experimentally observed dependence on q_{min} variations. This is confirmed by Fig. *7.* (bottom), which refers to such phase for the same simulations.

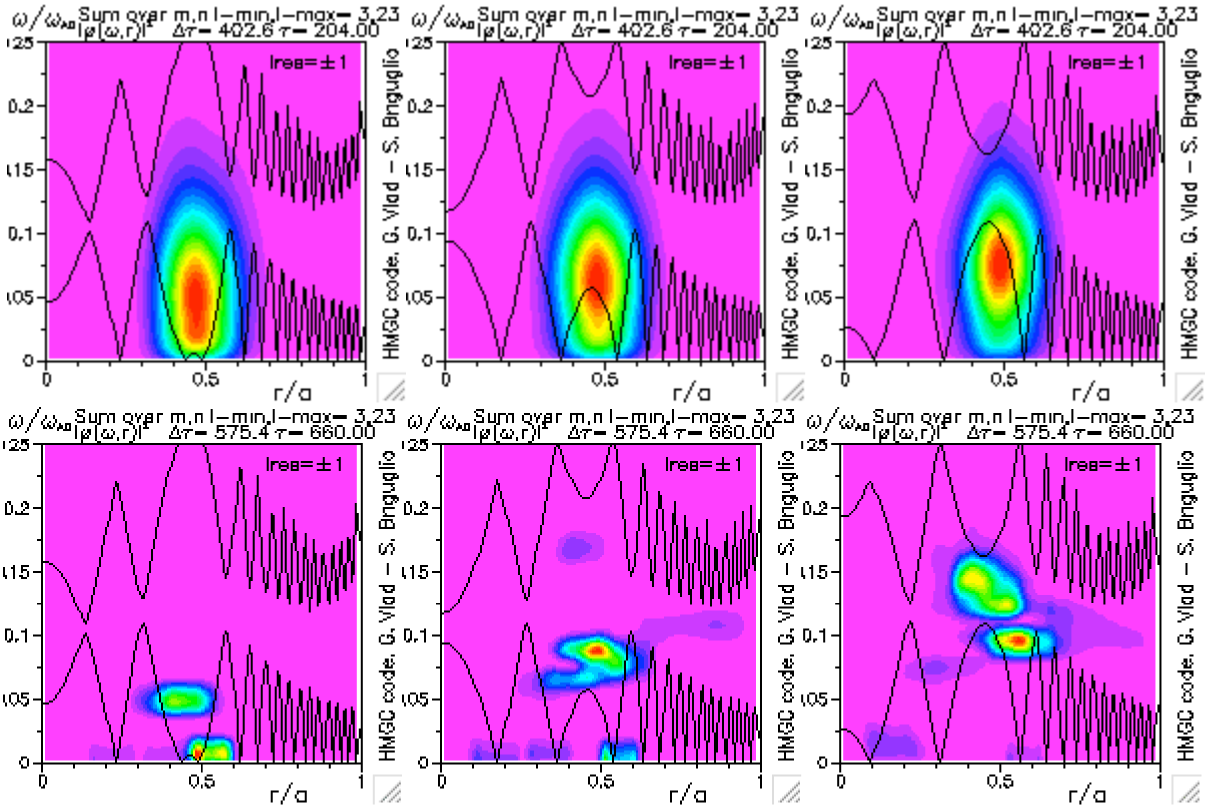

FIG. 7.: RSAE frequency spectra for $n = 2$ *and* $q_{min} = 3.99$ *(nominal value, left),* $q_{min} = 3.89$ *(center),* $q_{min} = 3.79$ *(right), for TRANSP energetic particle density profile. Top: linear phase. Bottom: saturated phase.*

From a quantitative point of view, the comparison between simulation and experimental frequency results is less satisfactory. This can be seen from Fig. *1.*, illustrating experimental measurements, and Fig. *8.*, where the frequencies of the saturated modes are reported, in physical units, versus their radial localization. A significant discrepancy between simulation results and experimental data is apparent: the absolute values of mode frequencies obtained in our simulations are lower than the experimental ones, also showing a larger spread with respect to the toroidal mode number n.

Several factors can be invoked to explain such discrepancy. The first one is given by the discussed dependency of the frequency on the details of the q profile. A little difference in the MSE-based evaluation of q would imply a large difference in the saturated mode frequency resulting from the simulation. Indeed, Fig. *9.* shows the frequencies of the saturated modes versus their radial localization for simulations analogous to those considered in Fig. *8.*, but for a lower-q_{min} equilibrium ($q_{min} \approx 3.89$).

A second reason of disagreement could be traced back to the fact that the MHD model our code

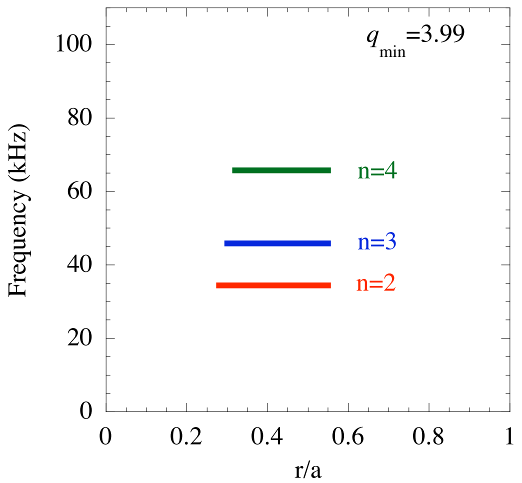

FIG. 8.: Frequency and radial localization of the dominant q_{min} modes for $n = 2$ (red), $n = 3$ (blue), and $n = 4$ (green), in physical units.

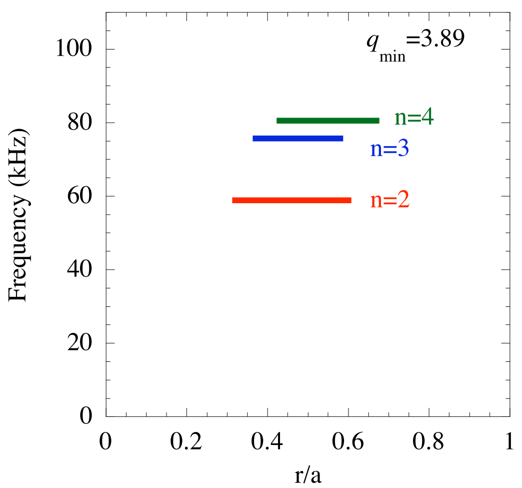

FIG. 9.: Same as Fig. 8. for simulations with a lower-q_{min} equilibrium ($q_{min} \approx 3.89$).

is based on ($O(\epsilon^3)$ reduced MHD equations in the zero-$\beta_{\rm bulk}$ approximation) neglects sound waves and geodesic curvature couplings. Significant modifications of the Alfvén continuum structure are expected when these effects are included: besides opening a new gap in the low frequency range (see, e.g., Ref. [9]) where discrete modes called Beta induced Alfvén Modes (BAEs) could exist, an up-shift of the toroidal gap and, possibly, of unstable mode frequencies would be obtained.

Accounting for experimentally observed plasma rotation $f_{\rm rot}$ (not considered in Fig. *8.*), does not seem, instead, sufficient to recover a good agreement. Indeed, this would yield a larger spread with n, the correction from rotation being proportional to the toroidal mode number itself ($f_{\rm lab} \approx f_{\rm mode} + nf_{\rm rot}$, where $f_{\rm lab}$ is the frequency observed in the laboratory frame and $f_{\rm mode}$ the frequency of the mode in the plasma frame). Moreover, Doppler shift corrections seem to be minor ones: rotation frequency at the q_{min} radius is $f_{\rm rot} \approx 1.86$ kHz.

Another interesting quantitative comparison between simulation and experiment concerns the amplitudes of the perturbed magnetic field. Typical values observed experimentally from magnetic probes are in the range $\delta B/B \approx 1 \div 5 \times 10^{-4}$. The values obtained in the saturated phase of the $n = 2$ TRANSP-profile simulation considered in Fig. *4. left* (nominal q profile) range from $\delta B/B \approx 1.8 \times 10^{-3}$ at the $r/a \approx 0.45$, where the perturbation is maximum, to $\delta B/B \approx 1.4 \times 10^{-4}$ at $r/a \approx 0.9$, which can be identified with the edge, as the HMGC simulations assume a perfect conducting wall at the plasma boundary. These values appear to be in reasonable agreement with the experimental measurements.

4. Conclusions

Particle simulations of Alfvénic activity in DIII-D tokamak reversed-shear discharges heated by neutral beams, performed by the HMGC code, are in fair agreement with experimental results. Several features of the experimental findings are quite well reproduced, in spite of several approximations intrinsic to the model underlaying the simulation code. In particular, Alfvén modes are found to be destabilized around the minimum-q radial position. Their frequency, in the saturated state, appears to be very sensitive to the details of the q profile (and, specifically, to the q_{min} value), consistently with the nature of the observed RSAE [1]. The values of the fluctuating magnetic field amplitude obtained in the simulations seem to be in reasonable agreement with experimental measurements as well.

Worse agreement is found in the absolute values of the mode frequencies, but it can be explained both by the fact that HMGC simulations, based on a zero-$\beta_{\rm bulk}$ model, do not include Beta induced Alfvén Mode physics and to the imprecision in determining the exact value of q_{min} for the equilibrium to be simulated.

Another significant output of the present simulation-based investigation is represented by the possible explanation of the large differences between the energetic particle density profile calculated by the TRANSP deposition code and that actually observed in the experiments, as inferred from the equilibrium reconstruction and FIDA measurements. Simulations referring to an initial state characterized by the former profile show that such state would give rise to the growth of very unstable Alfvén modes. Saturation of these modes would redistribute the energetic particles, flattening their density profile until the lower magnitude of density gradient observed in the experiments is reached. This Alfvénic activity can then be identified as the responsible for the quoted difference, although, at the present stage of investigation, it cannot be ruled out that the observed profiles can result from a succession of states close to marginal stability, rather than the fast saturation of a strongly driven mode.

References

[1] VAN ZEELAND, M. A. et al., Physical Review Letters **97** (2006) 135001.

[2] HEIDBRINK, W. W. et al., Alfvén instabilities in DIII-D: Fluctuation profiles, thermal-ion excitation, and fast-ion transport, in *Fusion Energy 2006 (Proc. 21st Int. Conf. Chengdu, 2006)*, CD-ROM file EX/6-3 and http://www-naweb.iaea.org/napc/physics/fec/fec2006/html/index.htm, Vienna, 2006, International Atomic Energy Agency.

[3] BUDNY, R. V. et al., Nuclear Fusion **35** (1995) 1497.

[4] LAO, L. L. et al., Nuclear Fusion **25** (1985) 1611.

[5] BRIGUGLIO, S. et al., Physics of Plasmas **2** (1995) 3711.

[6] STIX, T. H., Plasma Physics **14** (1972) 367.

[7] BRIGUGLIO, S. et al., Physics of Plasmas **14** (2007) 055904.

[8] BERK, H. L. et al., Physical Review Letters **87** (2001) 185002.

[9] CHU, M. S. et al., Physics of Fluids B: Plasma Physics **4** (1992) 3713.