Nonlinear Dynamics of Shear Alfvén Modes and Energetic Ion Confinement in Optimized Shear Tokamak Equilibria

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The dynamics of shear Alfvén modes in auxiliary-heated or close-to-ignition Tokamaks characterized by non-monotonic optimized $q$-profile equilibria (with $q$ being the safety factor) is an important issue in the current controlled-fusion research because of the relevance that such profiles can have in obtaining improved confinement regimes. The particular shape of the $q$ profile contributes to determine the Alfvén mode behavior in these equilibria, as it affects both the toroidal gap and the resonant-excitation frequencies, the size of the energetic-ion orbits, the radial extension and the toroidal coupling of the different poloidal harmonics as well as the energetic-ion drive intensity. Such a topic has recently attracted major interest because of the experimental measurements performed at JET during the current ramp-up phase of Reversed Shear discharges, characterized by hollow $q$ profiles [1], and the envisaged utilization of such improved confinement regimes for Next Step burning plasma experiments.

The nonlinear dynamics of shear-Alfvén modes in the presence of non-monotonic $q$ profiles are analyzed with regard to both saturation mechanisms and energetic particle transport by using the Hybrid MHD-Gyrokinetic simulation Code (HMGC) [2]. HMGC is a nonlinear initial-value hybrid code in which the bulk plasma is described by $O(\epsilon^3)$, low-$\beta$, reduced-MHD fluid equations, (with $\epsilon \equiv a/R_0$, $a$ and $R_0$ being the minor and major radius of the torus, respectively, and $\beta$ the plasma pressure normalized to the magnetic pressure), and the energetic particle population is described by the nonlinear gyrokinetic equations. The coupling between the two components (bulk plasma and energetic particles) is accounted for by the divergence of the energetic-particle pressure tensor included in the MHD vorticity equation [2].

A deeply hollow $q$ profile equilibrium ($q_0 \approx q_a \approx 5$, $q_{min} \approx 2$, $r_{q_{min}}/a \approx 0.5$ and $\epsilon = 0.1$, with $r$ being the radial coordinate) and a flat bulk plasma density profile are assumed in the following simulations. A single toroidal mode number, $n = 4$, is considered. The energetic – “Hot” – particle population is assumed to be characterized by an initial isotropic Maxwellian distribution function with constant temperature, Larmor radius $\rho_H/a = 0.01$, thermal speed $v_H/v_A = 1$, mass $m_H/m_i = 1$, with $v_A$ being the Alfvén velocity and $m_i$ the thermal-ion mass.

The first case we investigate corresponds to an initial $\beta_H$ profile with radial dependence close to the one measured on JET Reversed Shear discharges [1]. It is shown in
Fig. 1 top-left (solid line), and it is characterized by an on-axis value $\beta_{H,0} \approx 0.01$ and a peaking factor $\beta_{H,0}/\langle \beta_H \rangle \approx 3.7$ (with $\langle \ldots \rangle$ representing the volume average). Although the drive (proportional to $\alpha_H \equiv -R_0 q^2 \beta_H^\prime$, with “prime” denoting derivation with respect to $r$) is peaked around $r/a \approx 0.3$, the simulation shows a mode growing, during the linear phase, around the minimum-$q$ surface (Fig. 1 middle-left). We cannot rule out that other modes (e.g., close to the $\alpha_H$ peak) are unstable [3], but it is apparent that, under these conditions, the gap mode, though more weakly driven, suffers a lesser damping, resulting in the most unstable mode. The mode then saturates without changing appreciably its localization in space and frequency (see Fig. 1 bottom-left). The energetic particle profile is substantially preserved too by the saturation phase (Fig. 1 top-left, dashed line), which apparently occurs through a weak wave-particle trapping mechanism [4]. It is interesting to note that these results fit well with the experimental ones obtained at JET, where the Alfvén cascades have been explained [5] in terms of adiabatic frequency changes of gap modes located at the minimum-$q$ surface, in the presence of slow time evolution of the equilibrium $q$ profile.

In the second simulation, we assume a different, more peaked, initial $\beta_H$ profile (Fig. 1 top-centre, solid line), characterized by the same $\langle \beta_H \rangle$ value as in the first case, but a larger on-axis value ($\beta_{H,0} \approx 0.021$, corresponding to a peaking factor $\beta_{H,0}/\langle \beta_H \rangle \approx 7.8$). The $\alpha_H$ maximum does not change significantly its localization in comparison with the previous case, but its value is now much larger than the $\alpha_H$ value at the minimum-$q$ surface. Correspondingly, the linear phase of the simulation is dominated by a fast-growing resonant EPM [6] at the radial position where $\alpha_H$ is maximum, with real frequency in the upper continuum (Fig. 1 middle-centre). This mode saturates on a very short time scale (related to the inverse growth rate), by radially redistributing the energetic ions (Fig. 1 top-centre, dashed line). Following the displaced source, thus confirming its resonant character, the mode merges into a gap mode at the minimum-$q$ surface (Fig. 1 bottom-centre). By comparing final $\beta_H$ profiles and frequency spectra of the two examined cases (the first-case $\beta_H$ profile is also reported in Fig. 1 top-centre, by a dotted line), we note that the asymptotic states – corresponding, in fact, to the experimentally observable ones – are almost equivalent.

The third simulation assumes, for the initial $\beta_H$ profile, the same shape as in the second case, but a lower on-axis value: $\beta_{H,0} \approx 0.0085$. This profile is represented by the solid line in Fig. 1 top-right. The linear phase of the simulation is still dominated by the internal resonant EPM (Fig. 1 middle-right). After such mode saturates, a weaker gap mode at the minimum $q$ position becomes observable. Both modes survive asymptotically in time, at comparable saturation level (Fig. 1 bottom-right). Saturation takes place
through weak wave-particle trapping mechanisms and it leaves the energetic-particle radial distribution almost unchanged (Fig. 1 top-right, dashed line).

Note that some caution should be used when comparing HMGC results with a real experiment. The choice of isotropic Maxwellian distribution function (instead of, e.g., the anisotropic distribution function produced by ICRH) does not allow us to obtain a reliable determination of the $\beta_H$ thresholds for the occurrence of the different regimes evidenced by the simulations. Moreover, plasma shaping effects and fine tuning of the Alfvén continuum (via, e.g., bulk-plasma density profile) could modify the MHD properties of the bulk plasma and the associated damping mechanisms. Finally, smaller aspect-ratio values should be assumed for a better quantitative approximation of the energetic-particle drive. Nevertheless, we expect that the essential features of the saturation processes described in the paper would survive to a more detailed investigation.

We can then conclude that a fairly broad energetic-ion profile (like that measured on JET Reversed Shear discharges) with a gap mode localized around the minimum-$q$ surface can be obtained as the effect of a broad energetic-ion deposition profile (due, e.g., to auxiliary heating power density and collisional relaxation), weakly or not at all altered by the Alfvén-mode dynamics. It can also, however, be the effect of a more peaked deposition profile, deeply altered by the collective effects induced by resonant EPMs strongly driven, within the minimum-$q$ surface, by a large enough value of $\beta_H'$. An interesting indication for future experiments derives from such a conclusion. Energetic-ion deposition profiles should be compared with measured ones. If significant discrepancies are observed (with the measured profile being broader than that expected from power deposition), it can be the sign of the action of an internal resonant EPM (whose dynamics is however hidden to the experimental measurements by the very short time scales on which saturation and transport occur). To check this conjecture (and, possibly, to directly reveal the internal EPM), it is worth to perform lower-$\beta_{H,0}$ discharges, with the same shape of the energetic-ion deposition profile. If the dynamics we described in the present paper is the relevant one, one should observe the measured energetic-ion profile approaching the deposition one. At the same time, the radially resolved frequency spectrum should show the coexistence of the minimum-$q$ surface gap mode and the internal EPM, with comparable fluctuation levels.

References

Figure 1: Initial (solid lines) and final (dashed lines) $\beta_H$ profiles (top). Power spectra of the scalar potential fluctuations in the plane ($r/a$, $\omega/\omega_{A,0}$), with the Alfvén continuum structure (solid curves) superimposed, during linear (middle) and saturated (bottom) phases. Three cases are shown with different initial $\beta_H$ profiles: broad (JET-like) profile with $\beta_{H,0} \approx 1\%$ and $\langle \beta_H \rangle \approx 0.27\%$ (left); peaked profile with $\beta_{H,0} \approx 2.1\%$ and the same value of $\langle \beta_H \rangle$ (center); peaked profile with $\beta_{H,0} \approx 0.85\%$ and $\langle \beta_H \rangle \approx 0.11\%$ (right). We have defined the normalized time $\tau \equiv \omega_{A,0} t$, with $\omega_{A,0} \equiv v_A/R_0$. Note that the final $\beta_H$ profile for the JET-like case has been reported (dotted line) also in the top-center frame, for comparison.