

High- n Ideal TAE Stability of ITER

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Abstract. The ideal MHD stability analysis of high- n (n is the toroidal mode number) Toroidal Alfvén Eigenmodes (TAE's) is presented, using realistic and completely general ITER equilibria with shaped, up-down asymmetric, magnetic flux surfaces. An approach has been used, based on analytical-theoretical methods, which can give interesting results and allows us to analyze the conditions for enhanced TAE damping (although preventing us from computing the excitation thresholds). The frequency spectrum of TAE modes is found by solving the fully two dimensional problem using a two spatial-scale WKB formalism. The phase space integration is extended to a complete periodic orbit (at fixed frequency ω) in the (r, θ_k) phase-space (r is here a general flux coordinate and θ_k is the WKB eikonal entering in the expression of the radial *envelope* of the mode). The equilibria, analyzed here, are characterized by ideal TAE's localized in the half outer part of the plasma column, where the α -particle drive is expected to be small and modes are likely affected by continuum damping.

1. Introduction

It can be shown that the most unstable Alfvén modes in a tokamak reactor will be typically characterized by mode numbers n in the range $1 \ll \varepsilon a / \rho_{LE} \lesssim n \lesssim a / \rho_{LE}$, with $\varepsilon \equiv a / R_0$ (a and R_0 being, respectively, the minor and major radius of the torus) and ρ_{LE} the energetic particle (e.g., α -particles) Larmor radius [1]. The energetic particle drive is, in fact, proportional to the pressure gradient, and the instability growth-rate scales with the diamagnetic drift frequency $\omega_{*pE} \propto n$: modes with high mode number n will then be favorite. The range of toroidal mode numbers expected to be significant in an tokamak reactor derive from having Larmor radius and drift orbit widths of the energetic particles smaller or approximately equal to the poloidal wavelength ($\rho_{LE} \lesssim k_\theta^{-1} \approx a/n$, to avoid mode-particle detuning) and simultaneously larger or comparable with the radial wavelength ($\rho_{LE} \gtrsim k_r^{-1} \approx \varepsilon a/n$) [1] to maximize the ballooning-interchange drive due to the geodesic curvature term. The large mode number n creates serious resolution problems for conventional numerical codes, whereas an approach based on analytical-theoretical methods can overcome these difficulties. Previous analyses, using a *2D-WKB* code [2], have studied this problem either for (s, α) model equilibria (s is the magnetic shear and $\alpha = -q^2 R_0 \beta'$) and, more recently, for realistic equilibria [3]. In the former case, the simple equilibrium model allowed us to retain the details of the energetic particle dynamics, and thus, to explore the destabilization mechanism due to wave particle interactions. In the latter case, a more complete description of the equilibria was studied instead, at the price of neglecting energetic particle drive and all wave-particle resonant interactions; i.e., only marginally stable ideal global modes have been studied: this is the case considered here for the RTO/RC-ITER. Previous analyses have shown that TAE's can be shifted downward in frequency by finite- β effects and out of the toroidal frequency gap in the Alfvén continuum. As a consequence, the so-called continuum damping strongly increases, and these modes are expected to be stable also in presence of an energetic particle drive. The ideal MHD stability analysis, performed here, using a realistic equilibrium, can give interesting results and allows us to analyze the conditions for enhanced TAE damping (although preventing us from computing the excitation thresholds).

2. Theoretical Model

Here, frequencies are normalized to the Alfvén frequency on axis $\omega_A \equiv B_0/R_0\sqrt{4\pi\rho_0}$, and a plasma density profile $\rho = \rho_0\sqrt{1-\psi}$ has been assumed.

The frequency spectrum of TAE modes is found by solving the global dispersion relation [4]

$$\oint d\theta_k nq[r(\theta_k; \omega)] = \pi(2l + w) , \quad (1)$$

where $\theta_k \equiv k_r/nq'$ is the WKB eikonal entering in the expression of the radial *envelope* of the mode [4] (k_r is the radial wave vector, q is the safety and the prime ' indicates derivation with respect to the (radial) flux coordinate r), l is the radial mode number, w is the Maslov index (defined in the following) and ω is the mode frequency. The WKB eikonal θ_k is a function of the radial position, as it may be obtained from the solution of the local TAE dispersion relation

$$F(r, \theta_k; \omega) = 0 , \quad (2)$$

which is parameterized by the mode frequency ω . Furthermore, the integration in Eq. (1) for solving the global dispersion relation is extended to a complete periodic orbit (at fixed ω) in the (r, θ_k) phase-space, and w is either $w = 0$ for phase-space rotations or $w = 1$ for phase-space oscillations. Incidentally, we note that, in the up-down asymmetric equilibria considered here, $\theta_k = 0$ and $\theta_k = \pi$ are not WKB turning points – as in the general symmetric case – and that turning point positions need to be determined numerically from Eq. (2) with the condition $\partial F/\partial \theta_k = 0$.

3. ITER Equilibria

Here, a method of solution different from that used in ref. [3] has been used. In particular, a map of the function $\omega = \omega(r, \theta_k)$, as obtained from the solution of the local dispersion relation, Eq. (2), has been calculated. Typically, the function $\omega = \omega(r, \theta_k)$ is multivalued, and several surfaces can be identified in the space (r, θ_k, ω) (see, e.g., FIG. 1, where two of such surfaces are reproduced together with the surfaces corresponding to the Alfvén continua).

The global dispersion relation, Eq.(1), is obtained by integrating along the equi- ω contours in the plane (r, θ_k) , shown on the top of FIG. 1. The equilibria analyzed so far (standard monotonic q -profile RTO/RC-ITER IAM scenarii as provided by the ‘‘Plasma Equilibrium and Control Group, Plasma and Field Control Division, ITER EDA’’) are characterized by ideal TAE’s which are localized in the half outer part of the plasma column (see, e.g., FIG. 2, where the ideal MHD global TAE mode frequency spectrum and radial localization are shown for $n = 20$ and the equilibrium *sob13.3-111-b085-iam.1*, corresponding to the two surfaces $\omega = \omega(r, \theta_k)$ shown in FIG. 1). Several modes have at least one turning point very close to the (upper or lower) Alfvén continuum spectrum and, thus, are likely to suffer strong continuum damping. Other modes are well inside the toroidal gap and, thus, in principle could be easily driven unstable.

It is interesting to determine the toroidal mode number n above which the most dangerous modes (i.e., the ones which lie well inside the toroidal gap) appear. Focalizing on the modes shown in FIG. 1, solution ω_a , the very localized mode at $\omega/\omega_A \approx 0.62$ (radial mode number $l = 19$ for $n = 20$, see FIG. 2, and corresponding to a phase-space rotation $w = 0$) appears for toroidal mode number $n \geq 17$. The radially extended mode at $\omega/\omega_A \approx 0.6$ (radial mode number $l = -1$

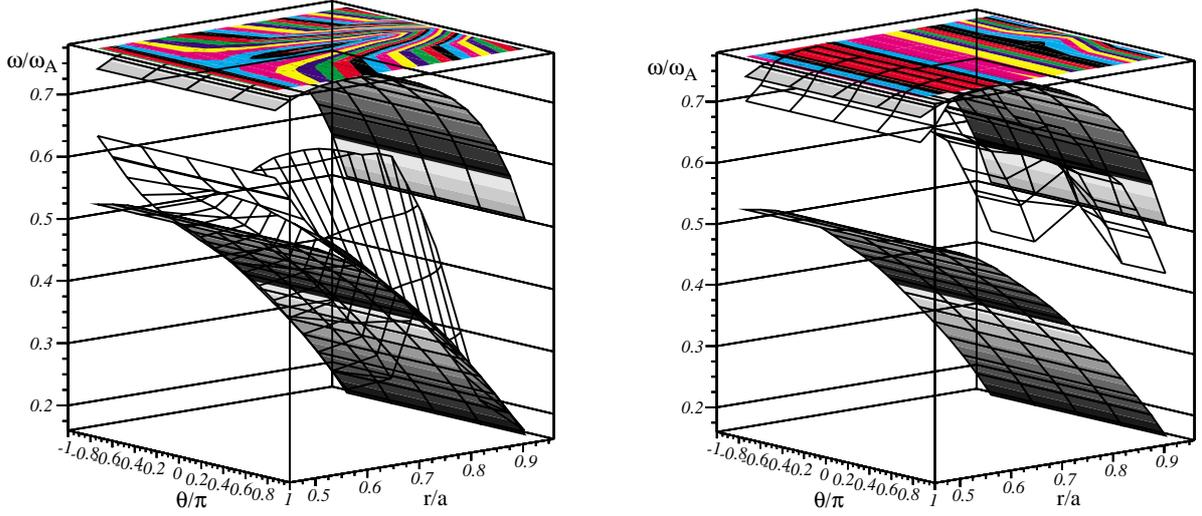


FIG. 1: The surfaces representing two solutions $\omega_a = \omega(r, \theta_k)$ (left) and $\omega_b = \omega(r, \theta_k)$ (right) (transparent grid and colored contour plot of it on top) as obtained by Eq. (2), for a portion of radius $r = 0.45 \div 0.96$, and $\theta_k/\pi = -1 \div 1$. Lower and upper continua are represented by gray surfaces.

for $n = 20$, see FIG. 2, and corresponding to a phase-space oscillation $w = 1$) appears for $n \geq 9$. Finally, let us consider the modes with lower frequencies ($0.44 \lesssim \omega/\omega_A \lesssim 0.57$, radial mode number $l = 23 \div 34$ for $n = 20$, see FIG. 2, and corresponding to phase-space rotations $w = 0$): these modes appear for $n \geq 2$ ($n \geq 4$ if modes with turning points clearly detached from the lower continuum ($\omega/\omega_A \gtrsim 0.54$) are considered) and become denser and denser in frequency as the toroidal mode number increases. For $n = 10$ already, the difference in frequency between modes with different radial mode numbers is $\omega/\omega_A \leq 0.02$. It has to be noted, nevertheless, that all these modes exist at a radial position where the energetic α -particle drive is expected to be small.

On the other hand, the gap structure of the reversed shear equilibria for similar scenarii are characterized by a much narrower extension in radius and, thus, by TAE's (if they exist) with a smaller radial extension.

References

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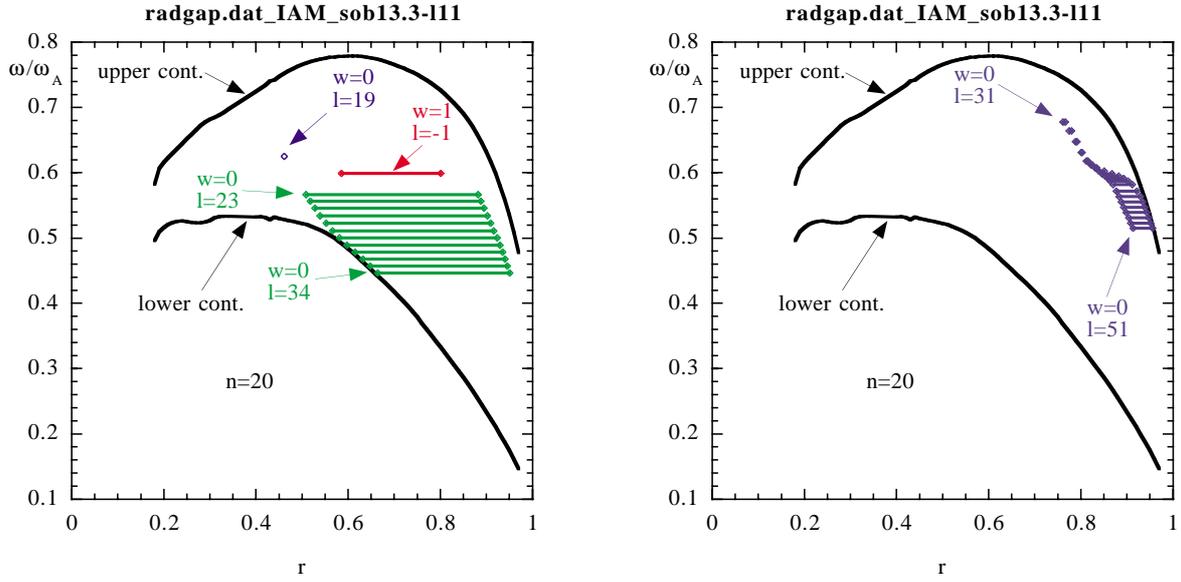


FIG. 2: Ideal MHD global TAE mode frequency spectrum for $n = 20$ and the equilibrium sob13.3-111-b085-iam.1 and relative, respectively, to the surface ω_a (left) and ω_b (right) (see FIG. 1). The radial mode number l , the Maslov index w and the toroidal gap boundaries are also shown.

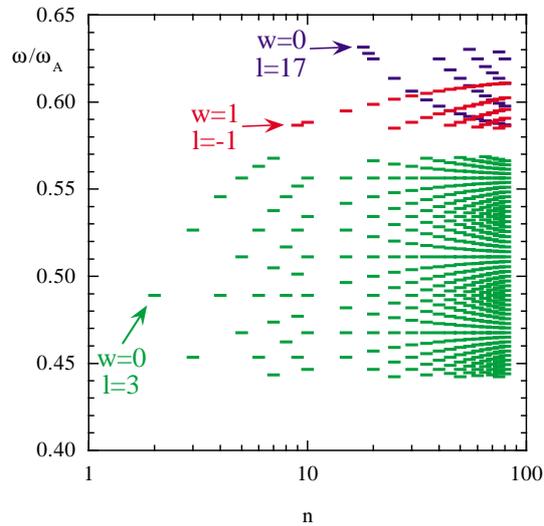


FIG. 3: Ideal MHD global TAE mode frequency spectrum as a function of the toroidal mode number n for the equilibrium sob13.3-111-b085-iam.1 and relative to the surface ω_a (see FIG. 1). The Maslov index w and the radial mode number l as the mode appears are also shown.