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The resistive damping of the toroidicity induced Alfvén eigenmode

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The MHD equations have been used to study the resistive damping of the toroidicity induced Alfvén eigenmode by employing the ballooning formalism. In the parameter regime $\nu \ll \epsilon^3$ (where ν is the normalised resistivity and ϵ is the inverse aspect ratio) the equations are solved by employing asymptotic techniques to yield a damping rate scaling as $\gamma \sim \nu/\epsilon^2$. For $\nu \gg \epsilon^3$, the damping rate is independent of ϵ and scales as $\gamma \sim \nu^{1/3}$. The analytical results have been verified numerically using a toroidal resistive MHD stability code.

Studies on toroidicity induced Alfvén eigenmodes [1–9] (TAE) are of current interest, because it is believed that these global MHD modes could cause anomalous alpha particle losses [7,8], consequently affecting the tokamak reactor operations. The TAE modes can exist in the frequency gaps which are between the shear Alfvén continuum bands. The gaps are the result of the coupling between neighbouring Fourier harmonics of the electromagnetic field perturbation, caused by nonuniformities in the toroidal magnetic field. The stability of the TAE modes are sensitively dependent on the balance between the destabilisation produced by the high energy particles and the continuum [4,5,9] and/or resistive damping. When the plasma resistivity is considered, several new branches of resistive shear Alfvén modes are created. Among them is the TAE mode, which is shown to approach the marginally stable ideal gap mode as $\gamma \sim \eta \exp(-\frac{1}{2}i\pi)$ [1] where γ is the damping rate and η is the resistivity. The damping of the TAE has been studied by Mett and Mahajan [6] using a kinetic analysis. Using a variational approach, they have examined the damping of the TAE due to the coupling with a kinetic Alfvén wave (KAW) and due to resistivity. In this Letter we investigate the effect of resistivity on the dynamics of the TAE mode, using a different approach, namely the asymptotic matching method (preliminary results have been published in ref. [10]). Our analytical findings are compared with the numerical results of the toroidal, resistive, full-MHD stability code MARS [11].

The starting point of our investigation is the high n resistive ballooning equation obtained in ref. [1], for an axisymmetric, large aspect ratio, low β toroidal plasma. Setting

$$\Phi = \frac{(1+s^2\theta^2)^{1/2}\phi}{[1+i\nu(1+s^2\theta^2)/\Omega]^{1/2}}$$

in eq. (24) of ref. [1] we obtain

$$\frac{d^2\Phi}{d\theta^2} + \Phi\left(\frac{s^2[1+(i\nu/\Omega)(1-3s^2\theta^2)(1+s^2\theta^2)]}{(1+s^2\theta^2)^2[1+i\nu/\Omega(1+s^2\theta^2)]^2} + \Omega^2[1+2\epsilon\cos(\theta)]\frac{1+i\nu(1+s^2\theta^2)}{\Omega}\right)=0, \quad (1)$$

where ϕ is the electrostatic potential, $\epsilon = 2r/R$, $s = nq'/q$, $\nu = \eta(nq/r)^2/\omega_A$, $\Omega = \omega/\omega_A$, $\omega_A = V_A/qR$, $V_A^2 = B_0^2/\rho$, and R, r, θ, ζ are the major radius, minor radius, poloidal angle and toroidal angle respectively. In the ideal limit ($\nu = 0$) eq. (1) is essentially a Mathieu equation whose characteristic values of Ω^2 as a function of ϵ define the frequency gaps between the continuum bands. For large θ, the solutions we are interested in are exponentially growing or decaying and the effect of resistivity for large θ

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can be studied by performing the usual two scale analysis.

Letting $\Phi=\psi_c\cos(\frac{1}{2}\theta)+\psi_s\sin(\frac{1}{2}\theta)$, where ψ_c and ψ_s have slow variations, eq. (1) for $\theta\gg 1$ becomes

$$\frac{d^2y}{d\theta^2}+y\left(\frac{i\nu s^2\Gamma_-/4\Omega-1/2(i\nu s^2\theta/2\Omega)^2}{(\Gamma_-+i\nu s^2\theta^2/4\Omega)^2}\right.$$

$$\left.+(\Gamma_++i\nu s^2\theta^2/4\Omega)(\Gamma_-+i\nu s^2\theta^2/4\Omega)\right)=0 \quad (2)$$

where

$$y=\psi_c(\Gamma_-+i\nu s^2\theta^2/4\Omega)^{-1/2}, \quad \Gamma_\pm=\Omega^2(1\pm\epsilon)-\tfrac{1}{4}.$$

Setting $\Omega\sim 1$, $\Gamma_\pm\sim\epsilon$, $s\sim 1$, $s\theta\sim\epsilon^{-1}$ and balancing $\nu s^2\theta^2/\Omega$ with Γ_- in eq. (2), we arrive at the optimal ordering for ν, $\nu\sim\epsilon^3$. In order to solve for Ω inside the gap, it is well known that the asymptotic matching methods can be employed by considering two regions of $s\theta$, $s\theta\sim 1$ and $s\theta\sim\epsilon^{-1}$. The solution of eq. (2) should vanish asymptotically as $|\theta|\to\infty$ and, for small values of $s\theta$, match smoothly with the solutions in the region $s\theta\sim 1$. The solution in the transition region $1\ll s\theta\ll\epsilon^{-1}$ is given by

$$y\sim 1+b_0\Gamma_-s\theta, \quad (3)$$

where b_0 is a constant which is independent of ϵ and is obtained by matching the solutions to the region $s\theta\sim 1$. An explicit expression for b_0 can be obtained for the case of high toroidal mode number, whilst for low toroidal mode number b_0 must be numerically obtained. Since eq. (2) cannot be solved exactly, we examine the solutions in the asymptotic regimes, $\nu/\epsilon^3\ll 1$ and $\nu/\epsilon^3\gg 1$. In the former parameter regime, the modification produced by resistivity on the gap mode is small and eq. (2) can be considerably simplified. The first term in the parentheses can be dropped and the quartic term in θ can be neglected as compared with the quadratic term in θ. With these simplifications, eq. (2) reduces to a parabolic cylinder equation. The solutions are given by

$$y=AU(a,x), \quad (4)$$

where a and x are given by

$$a=-\frac{\Gamma_-\Gamma_++i\nu s^2/4\Omega\Gamma_-}{[-i\nu s^2(\Gamma_-+\Gamma_+)/\Omega+(i\nu s^2/\Omega\Gamma_-)^2]^{1/2}},$$

$$x=\theta[-i\nu s^2(\Gamma_-+\Gamma_+)/\Omega+(i\nu s^2/\Omega\Gamma_-)^2]^{1/4}.$$

Carrying out the matching procedure, we obtain

$$-\sqrt{2}\frac{\Gamma(\frac{1}{2}a+\frac{3}{4})}{\Gamma(\frac{1}{2}a+\frac{1}{4})}\left[-\frac{i\nu s^2(\Gamma_-+\Gamma_+)}{\Omega}+\left(\frac{i\nu s^2}{\Omega\Gamma_-}\right)^2\right]^{1/4}$$

$$=b_0s. \quad (5)$$

The dispersion relation for $\nu\ll\epsilon^3$ is obtained by making a large argument expansion of the gamma functions in eq. (5), i.e. $\frac{1}{2}a\gg 1$. On solving for the damping rate γ, it is found that

$$\frac{\gamma}{\omega_A}=-\frac{2\nu(1+b_0^2s^2)}{b_0^2s^2\epsilon^2}, \quad (6a)$$

while Γ_-^2 is given by

$$\Gamma_-^2=\frac{\epsilon^2}{16(1+b_0^2s^2)} \quad (6b)$$

Equation (2) is now examined in the parameter regime $\nu/\epsilon^3\gg 1$. In this case resistivity alters the mode structure greatly and the mode may not even be in the gap. It is convenient in this case to adopt the following normalisation,

$$\delta\Omega=\Omega-\tfrac{1}{2}, \quad \lambda=\delta\Omega(\tfrac{1}{2}\nu s^2)^{-1/3},$$

$$t=\theta\lambda^{-1/2}(\tfrac{1}{2}\nu s^2)^{1/3}.$$

The resulting equation is

$$\frac{d^2y}{dt^2}+y\left(\frac{i(1-2it^2)}{(1+it)^2}+\lambda^3(1+it)^2\right)=0. \quad (7)$$

Equation (7) has been solved using standard WKB techniques. In the region $t\ll 1$, the expression in the brackets in eq. (7) can be expanded linearly to obtain a parabolic cylinder equation. On matching the solution of the parabolic cylinder equation in the small argument limit with that of the solution given by eq. (3) the following dispersion relation is obtained,

$$b_0s\lambda^{1/2}(\tfrac{1}{2}\nu s^2)^{-1/3}=-\frac{\sqrt{2}}{\alpha}\frac{\Gamma(\frac{3}{4}+\frac{1}{2}z)}{\Gamma(\frac{1}{4}+\frac{1}{2}z)}, \quad (8)$$

where $\alpha=(\frac{1}{8}i)^{1/4}\lambda^{-3/4}$ and $z=-\alpha^2\lambda^3$.

We solve eq. (8) for $\lambda\gg 1$. In this regime eq. (8) can only be satisfied when the argument of the gamma function of the denominator is equal to a negative integer,

$$\tfrac{1}{4}+\tfrac{1}{2}z=-p\,, \tag{9}$$

where p is a large integer. Of the three roots of λ obtained by solving eq. (9), it is seen that only the damped roots given by

$$\gamma/\omega_A=-\tfrac{1}{2}(8n^2)^{1/3}(\tfrac{1}{2}s^2)^{1/3}\nu^{1/3}$$

are acceptable by the boundary conditions (a similar result has been obtained in ref. [6]).

The analytical results have been verified using the toroidal, resistive, full-MHD stability code MARS [11]. In fig. 1 a plot of the real and imaginary parts of Ω versus ν is shown for two different values of ϵ, $\epsilon=0.032$ and $\epsilon=0.16$. The equilibrium used is characterised by $q_0=1.1$, $q_a=1.8$ and a circular cross section. The toroidal mode number is given by $n=1$ and the gap is located at $q=1.5$. For $\epsilon=0.032$ the two different regimes of the damping rate γ versus ν, i.e. $\gamma\sim\nu/\epsilon^2$ and $\gamma\sim\nu^{1/3}$, are clearly observed. For $\epsilon=0.16$, the transition region ($\nu\sim\epsilon^3$) shifts towards higher values of ν, and the scaling $\nu^{1/3}$ is only marginally recovered.

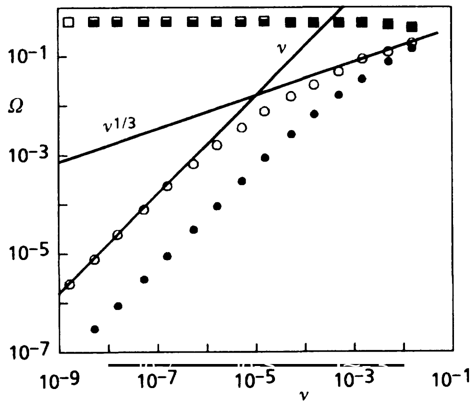

Fig. 1. Plot of Ω versus ν, the square symbols refer to the real part of Ω while the circles refer to the imaginary part of Ω. The open symbols (for both real and imaginary Ω) correspond to $\epsilon=0.032$, while the closed symbols correspond to $\epsilon=0.16$. The solid lines are the reference plots of $\nu^{1/3}$ and ν.

In conclusion, we have investigated the resistive damping of the TAE mode by using an optimal ordering of the normalised resistivity (ν) with the aspect ratio (ϵ), $\nu\sim\epsilon^3$. It is found that in the regime $\nu\ll\epsilon^3$, the damping rate γ is linearly proportional to ν/ϵ^2, whilst for $\nu/\epsilon^3\gg1$, γ is independent of ϵ and scales as $\nu^{1/3}$.

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