Interaction of runaway electrons with lower hybrid waves via anomalous Doppler broadening

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(Received 4 January 2002; accepted 30 January 2002)

Due to the relativistic decrease of the electron cyclotron frequency, a cyclotron resonance may appear between runaway electrons and lower hybrid waves. A single particle description of the runaway dynamics [J. R. Martín-Solís et al., Phys. Plasmas 5, 2370 (1998)] is extended to analyze the effect of the interaction of runaway electrons with lower hybrid waves via anomalous Doppler broadening. The conditions under which the resonant interaction can play a role in limiting the runaway energy are established and it is shown that, under typical lower hybrid current drive operation parameters, an efficient wave-particle coupling may occur. Observations of a fast pitch angle scattering event during the current decay phase of Ohmic discharges in the Toroidal Experiment for Technically Oriented Research (TEXTOR) [R. J. E. Jaspers, Ph.D. thesis, Technical University Eindhoven (1995)] are interpreted in terms of such interaction. © 2002 American Institute of Physics. [DOI: 10.1063/1.1470165]

I. INTRODUCTION

Electrons in a tokamak with energies higher than some critical energy are continuously accelerated by the toroidal electric field, i.e., they run away. The runaway electrons speed up very quickly until the synchrotron radiation losses in the curved path of the tokamak balances the energy gain in the electric field. There is concern about the damage that these energetic electrons might cause on the first wall structures, particularly during disruptions as the large electric fields induced during the current quench phase of a tokamak disruption can lead to a large number of runaway electrons with energies up to hundreds of MeV. For this reason, experimental and theoretical studies are being carried out in order to determine the conditions which can provide a reduction of the runaway damage to the machine.

The effect of the runaway electrons when they impinge on the vessel walls or plasma facing components is strongly dependent on the energy gained in the electric field. Therefore, it is of interest to investigate mechanisms that might help to control the energy that the runaway electrons can achieve. Any pitch angle scattering process, increasing the energy perpendicular to the magnetic field and the power radiated by the electron, can create a barrier to a further increase in the runaway energy. As an example, an upper boundary on the runaway energy can appear due to a resonance between the electron gyromotion and the magnetic-field ripple of the tokamak. This process has been described in detail in Ref. 2 and it has been predicted that the interaction with the magnetic-field ripple of the tokamak may set a limit on the maximum energy attainable by disruption generated runaway electrons in large tokamaks like the Joint European Torus (JET) or the planned International Thermonuclear Experimental Reactor (ITER).

It has been suggested that, due to the relativistic decrease of the electron cyclotron frequency, a resonance may occur between runaway electrons and lower hybrid (LH) waves in tokamak discharges which, under proper conditions, can efficiently convert parallel energy into perpendicular energy and give rise to the blocking of the runaway energy. In fact, evidence for such interaction has been claimed in the Frascati Tokamak (FT), where the disappearance of the photo-neutron production during the radio frequency heating has been explained by an anomalous Doppler resonance of relativistic runaway electrons with the lower hybrid waves, stopping the electron acceleration necessary to reach high enough energies for photo-neutron production. Also a fast pitch angle scattering process for the runaway electrons has been observed during the current decay phase of Ohmic discharges in the Toroidal Experiment for Technically Oriented Research (TEXTOR) which has been attributed to the interaction of the runaway electrons with lower hybrid waves excited by lower energy runaways in the so-called Parail Pogutse instability.

The purpose of this paper is to investigate the conditions under which the cyclotron resonance between runaway electrons and lower hybrid waves may be sufficient to cause a barrier on the electron energy during a tokamak discharge. The dynamics of the runaway electrons will be analyzed using a test particle description similar to that used in Refs. 2 and 9 but including the resonant interaction with the waves. The diffusion coefficient describing the interaction at the cyclotron resonance together with the single particle equations for the runaway dynamics will be presented in Sec. II. Indirect interaction due to LH current profile modification inducing perturbed runaway orbits will not be considered. As an

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example of application, the observations of fast pitch angle scattering events reported in TEXTOR will be interpreted in terms of the resonant interaction of the runaway electrons with lower hybrid waves. In Sec. III, the conditions under which this mechanism can efficiently block the electron energy will be determined, and the possibility of controlling the runaway energy for typical conditions during lower hybrid current drive operation will be analyzed. The conclusions will be presented in Sec. IV.

II. BASIC EQUATIONS

A. Wave–particle interaction

The lower hybrid wave is the slow branch of the cold dispersion relation, which can be studied in the electrostatic approximation. Under typical conditions during current drive, a Landau coupling of the wave is allowed to the parallel velocity of the suprathermal electrons. On the other hand, the perpendicular degree of freedom of the thermal electrons is not coupled to the LH wave as the electron thermal wavelength can be of the order of the Larmor radius, so that the increase in the Larmor radius, the wave frequency may be of the absolute value of the electron charge;

\[ E = E_x e_x + E_z e_z \]

\[ (E_x e_x + E_z e_z) \exp\{i(k_L x + k_L z - \omega t + \Phi)\}. \]

(2)

\( E_z \) and \( E_x \) are, respectively, the parallel and perpendicular components of the wave electric field, \( E || \) and \( E \perp \) their amplitudes, \( k || \) and \( k \perp \) the parallel and perpendicular wave numbers (for an electrostatic wave, \( E || / E \perp = k || / k \perp \)); \( \Phi \) is an angle which depends on the initial phase between the wave and the electron in its orbit. Thus, the components of the Lorentz force equation can be expressed as

\[ \frac{dp_x}{dt} + \omega_{ce} p_y = -e E_x, \]

(3)

\[ \frac{dp_y}{dt} - \omega_{ce} p_x = 0, \]

(4)

\[ \frac{dp_z}{dt} = -e E_z. \]

(5)

Focusing on the motion in the (x,y)-plane, multiplying Eqs. (3) and (4) by \( p_x \) and \( p_y \), respectively, and adding, we get

\[ \frac{1}{2} \frac{d}{dt} \left( \frac{1}{2} p_x^2 \right) = -e E_x p_x \]

(6)

\[ [p_\perp = (p_x^2 + p_y^2)^{1/2} \] is the perpendicular (to the magnetic field) electron momentum.

On the right hand side of Eq. (6) we use now the unperturbed orbit

\[ z = v_z t; \]

\[ (p_L = v_\perp / \omega_{ce} \] is the Larmor radius) and

\[ p_x = m_e \gamma \omega_y = p_L \cos \omega_{ce} t = \frac{p_L e^{i \omega_{ce} t} + e^{-i \omega_{ce} t}}{2}, \]

(8)

so that, taking into account Eq. (2), we can write

\[ \frac{dp_\perp}{dt} = -e E_\perp p_\perp (e^{i \omega_{ce} t} + e^{-i \omega_{ce} t}) \]

\[ \times e^{i(k_L p_L \sin \omega_{ce} t + k || \omega || t - \omega t + \Phi)} \]

\[ = -e E_\perp p_\perp \sum_n [J_{n+1}(k_L p_L) \]

\[ + J_{n-1}(k_L p_L)] e^{i(n \omega_{ce} t + k || \omega || t - \omega t + \Phi)}. \]

(9)

\( J_n \) are the Bessel functions of integer index resulting from the expansion \( \sum_n e^{i x} \sin \omega = \sum_n J_n(x)e^{i \omega} \).

According to Eq. (9), particles satisfying the condition

\[ n \omega_{ce} + k || v || - \omega = 0, \]

(10)

will be in resonance with the wave. Their response to the wave is not an oscillating momentum perturbation, but a steadily increasing change in its momentum. This means that, depending on the sign of the change in the momentum, they will either absorb energy from the wave or feed energy into it.
For runaway electrons \((v_j \sim c)\), the resonance condition (10) leads to \(n \nu_{we} = \nu(1 - N_j) < 0\), so that \(n > 1\), which must be less than zero. In such case \((n < 0)\), we have the anomalous Doppler resonance, and parallel electron energy is converted into perpendicular energy. The anomalous Doppler resonance, increasing the perpendicular electron energy, and therefore, the power radiated by the electron, may lead to a barrier to a further increase in the electron energy. We will be primarily interested in the lowest resonance numbers \((n = -1\) and \(n = -2)\) as, for larger \(|n|\) values, the electron energy is too high and a quasi-perpendicular Landau resonance due to the large electron Larmor radius \(k_\perp \rho_L > 1\) would take place, which will not be considered here.

Near the \(n\)th resonance the equation of motion (9) (keeping only the real part) can be simplified to

\[
\frac{dp}{dt} = -eE_\perp \left[J_{n-1}(k_\perp \rho_L) + J_{n+1}(k_\perp \rho_L)\right]
\times \cos\left[\left(n \nu_{we} + k_\| v_\| - \omega\right)t + \Phi\right]
\times \left[1 + \frac{p}{k_\| v_\|} \left(m_e \nu_{we} + k_\| v_\| - \omega\right)\right],
\]

where it has been used that \(J_{n-1}(x) + J_{n+1}(x) = 2J_n(x)/x\).

Following a similar procedure for the parallel momentum equation \(p_\| (p_\| = p_c)\) we get, from Eq. (5),

\[
\frac{dp_\|}{dt} = -eE_\| e^{i(k_\perp \rho_L \sin \nu_{we} t + k_\| v_\| t - \omega t + \Phi)}
\times \left[1 + \frac{p}{k_\| v_\|} \left(m_e \nu_{we} + k_\| v_\| - \omega\right)\right],
\]

and, close to the \(n\)th resonance

\[
\frac{dp_\|}{dt} = -eE_\| J_n(k_\perp \rho_L) \cos\left[\left(n \nu_{we} + k_\| v_\| - \omega\right)t + \Phi\right].
\]

Thus, the final set of equations describing the electron motion near a single cyclotron resonance will be

\[
\dot{p}_\perp = -eE_\perp \left[J_{n-1}(k_\perp \rho_L) + J_{n+1}(k_\perp \rho_L)\right] \cos(\delta),
\]

\[
\dot{p}_\| = -eE_\| J_n(k_\perp \rho_L) \cos(\delta),
\]

with \(\delta = n \nu_{we} + k_\| v_\| - \omega\).

As the runaways collide with the plasma particles, the gyration phase changes randomly and the wave field \(E_\perp\) accelerating the gyromotion imposes a random walk on the gyromomentum \(p_\perp\). Thus, the dynamics in the resonance of the runaway electrons may be simulated via a diffusion coefficient for the gyromomentum given by

\[
D_{p_\perp} = \frac{d\langle p_\perp^2 \rangle}{dt} = \pi(eE_\perp)^2 \left|J_n(k_\perp \rho_L)\right|^2 \left|\frac{k_\| v_\|}{k_\perp \rho_L}\right|^2,
\]

if \(\left|n \nu_{we} + k_\| v_\| - \omega\right| < \Delta w\),

\[
\Delta w = 2 \left[\frac{eE_\| \left|J_n(k_\perp \rho_L)\right|}{m_e \nu_{we}}\right]^{1/2},
\]

where

\[
\nu_{we} = \frac{n \nu_{we} + k_\| v_\| - \omega}{\Delta w}.
\]

The pitch angle scattering process taking place at the resonant region will also have an effect on the longitudinal component of the electron momentum. From Eq. (14), we immediately get

\[
\frac{dp}{E_\| \rho_L \nu_{we} = n} \frac{\delta p}{\delta p}.
\]

**B. Single particle description**

The normalized test relaxation equations describing the resonant interaction with the lower hybrid waves via anomalous Doppler broadening may be obtained taking account the pitch angle scattering given by Eq. (16) and the effect on the parallel electron momentum given by Eq. (18). These equations, including the effect on the runaway dynamics of the acceleration by the inductive toroidal electric field \(E_\phi\), the collisions with the plasma particles and the synchrotron radiation losses (see Ref. 9 for a detailed description of these effects) will take the form

\[
\frac{dq_\|}{d\tau} = -\gamma(\alpha + \gamma) q_\| \left[\left(F_{gc} + F_{gy} \frac{q_\|^2}{q_\perp^4}\right) \gamma^2 \left(\frac{v}{c}\right)^3 q_\parallel\right]
\]

\[
+ D_{LH} \gamma^{1/2} \left|J_n(k_\perp \rho_L)\right|^{3/2},
\]

\[
\frac{dq_\perp}{d\tau} = 2 \gamma(\alpha + \gamma) q_\| \left[\frac{3}{4} \gamma^2 - \gamma^2 \left(F_{gc} + F_{gy} \frac{q_\|^2}{q_\perp^4}\right) \gamma^2 \left(\frac{v}{c}\right)^3 q_\perp\right]
\]

\[
+ D_{LH} \gamma^{1/2} \left|J_n(k_\perp \rho_L)\right|^{3/2},
\]

with

\[
D_{LH} = \frac{\pi(eE_\|)^3 n^2 k_{\|}^2}{4 \nu m_e \gamma^{5/2} |A_{LH}|},
\]

if \(\left|n \nu_{we} + k_\| v_\| - \omega\right| < \Delta w\),

\[
0, \quad \text{otherwise},
\]

and \(\Delta w = 2 A_{LH} \gamma^{1/2} \left|J_n(k_\perp \rho_L)\right| \gamma^{1/2}\), where \(A_{LH}\) from Eq. (17), is given by
energy of the measured runaway electrons is in the range of \( \approx 23 \text{ MeV} \). It was proposed in Refs. 7 and 8 that this event involves a two stage process, starting with the excitation of lower hybrid waves via the Parail Pogutse instability, and the subsequent pitch angle scattering of the runaway electrons on the lower hybrid waves via the anomalous Doppler effect. The excited lower hybrid waves are peaked around the ion plasma frequency \( \omega_{pi} \), so that \( \omega \approx \omega_{pi} = \sqrt{Z_{eff} n_e/m_e c} \), which for typical TEXTOR parameters \( |N| = 4 \), \( Z_{eff} = 2 \), \( n_e = 0.5 \times 10^{19} \text{ m}^{-3} \) yields \( \omega \approx 2.9 \text{ GHz} \). Furthermore, Parail and Pogutse\(^{11} \) estimate for the waves that \( (k_i/k)^2 \approx 1/3 \). It can be observed in Fig. 1 that, initially, the electron follows the usual trajectory in momentum space but when the resonance is reached \( \left( q_\parallel = 46 \right) \), it experiences a strong pitch angle scattering which leads to large values of \( q_\perp \) and, therefore, to strong synchrotron losses. The electron energy vs time is plotted in Fig. 1(b), showing the energy blocking due to the resonant interaction. At the equilibrium point, \( P_n \), the electron pitch angle is \( \theta_n \approx 0.16 \text{ rad} \) and the electron energy \( \sim 23 \text{ MeV} \).

The dynamics of the interaction at the resonance may be described as follows. When the electron accelerated by the inductive electric field \( E_R \) enters the resonant region, a strong diffusion in momentum-space takes place converting its parallel energy into perpendicular energy. The equilibrium is reached at the resonance when \( \dot{q} = 0 \).

In a similar way to the runaway–ripple interaction described in Ref. 2, the equilibrium state is dynamical: \( \dot{q} = 0 \) but \( q_\parallel < 0 \) so that the electron is thrown out the resonance; then, when the particle is outside the resonant region, the inductive electric field pushes it again into the resonance, starting the process again, and resulting in the accumulation of the particle near the equilibrium point.

III. CONDITIONS FOR EFFICIENT INTERACTION

A. Lower electric field

We will investigate now the conditions under which the interaction with the nth cyclotron resonance can set a limit on the energy that the runaway electrons can reach.

The first condition arises from the requirement that the electron experiences the resonant interaction before the radiation limit is reached. Taking into account that in all cases of interest \( \cos \theta = 1 \) \( (u_\parallel = c) \), the resonant interaction will take place for an electron energy \( \gamma_n = |n| \Omega_n / (k/c - \omega) \). Thus, for an efficient interaction, the resonant energy \( \gamma_n \) should be less than the radiation limit \( \gamma_S \) (see Refs. 2 and 9 for details on \( \gamma_S \)). This means that there exists a minimum inductive electric field \( D_{low} \) (the lower electric field for efficient wave–particle interaction) for which the wave–particle interaction takes place: the electric field for which \( \gamma_S = \gamma_n \). It can be shown, using that in all practical cases \( \gamma_S > 1 \) and \( \cos \theta = 1 \), that

\[
D_{low} = \frac{1 + F_{gc} \gamma_n^4 + \sqrt{(1 + F_{gc} \gamma_n^4)^2 + 4 \alpha F_{gy} \gamma_n}}{2}
\]

[the derivation of Eq. (23) follows the same arguments used in Appendix A in Ref. 2].
FIG. 2. For the \( n = -1 \) resonance: (a) Equilibrium pitch angle \( \theta_n (\hat{q} = 0) \) and pitch angle \( \theta_\parallel (\hat{q} = 0) \) plotted vs the normalized inductive electric field (perpendicular wave field, \( E_\perp = 300 \text{ V/m} \)). The upper electric field \( D_{up} \) is indicated. (b) Upper electric field \( D_{up} \) vs perpendicular wave field \( E_\perp \). Plasma and wave parameters are the same as that in Fig. 1.

B. Upper electric field and critical pitch angle

The condition \( D > D_{\text{low}} \) does not guarantee the effective blocking of the runaway energy at the resonance. The electron will cross the resonance if, during the pitch angle scattering process at the resonant region, the condition \( \dot{q}_\parallel > 0 \) is found before the equilibrium state \( (\hat{q} = 0) \), i.e., if the characteristic time for crossing the resonance is less than the characteristic time to reach the equilibrium.

The two cases of interest, the resonances \( n = -1 \) and \( n = -2 \), are illustrated in Figs. 2 and 3, respectively. The plasma conditions and wave parameters are the same that in Fig. 1, corresponding to the observations of fast pitch angle scattering events in TEXTOR. Figure 2(a) shows the pitch angle \( \theta_n \) at the equilibrium point \( (\hat{q} = 0) \) versus \( D \) together with the pitch angle \( \theta_\parallel \) for which \( \dot{q}_\parallel = 0 \) at the \( n = -1 \) resonance and for a perpendicular wave field \( E_\perp = 300 \text{ V/m} \): if, for a given value of \( D \), \( \theta_\parallel < \theta_n \), the condition \( \dot{q}_\parallel > 0 \) will be reached before \( \hat{q} = 0 \) and the particle will cross the resonance. Thus, the results shown in Fig. 2(a), indicate that, for the case \( n = -1 \), the region for efficient interaction will be determined by an upper electric field \( D_{up} \), given by the minimum of \( D \) vs \( \theta_\parallel \). In Fig. 2(b), the electric field \( D_{up} \) calculated according to this criterion is plotted vs the perpendicular wave field \( E_\perp \), showing closely a linear dependence of \( D_{up} \) on \( E_\perp \). It is also inferred from Fig. 2(b) that, to explain the fast pitch angle scattering events observed in TEXTOR (inductive electric field \( \sim 0.1 \text{ V/m} \), \( D \sim 21.8 \)), a wave field \( E_\perp \) larger than \( 300 \text{ V/m} \) would be required. An analytical expression for \( D_{up} \) can be found in the small-Larmor-radius approximation \( (k\rho_L \ll 1) \) and is given by (see Appendix B for details):

\[
D_{up} \approx 1 + \frac{\alpha}{\gamma_n} F_{g_\perp}^{4} + 5 (F_{g_\perp}^{4} \gamma_n^{1/5}) \times \left[ \frac{\pi \Omega_0}{32 A_{\text{LH}} \nu} \left( \frac{e}{2 \Omega_0 m_c} \right)^{3/2} \right]^{4/5} E_\perp^{6/5},
\]

where \( \gamma_n \) is the energy at the \( n = -1 \) resonance, \( \gamma_n = \Omega_0 / (\kappa \rho_L - \omega) \), and \( A_{\text{LH}} = (1 + 1/N_r^{3/2})^{1/2} \). Equation (24) illustrates the linear dependence on \( E_\perp \), \( D_{up} \approx E_\perp^{1/2} \), previously discussed.

Application of this criterion for typical lower hybrid current drive conditions \([N_r = 2, N_r / |N_r| \approx E_\perp / |E_r| \approx 10, \text{ wave frequency} f = 2 - 10 \text{ GHz}]\) also indicates an efficient interaction at the \( n = -1 \) resonance. Thus, for the parameters reported in Ref. 5 \([\omega = 10 \pi \text{ GHz}, N_r = 2, N_r = 20, \text{ parallel wave electric field} E_\perp = 7.0 \text{ kV/cm, inductive electric field} \]

FIG. 3. For the \( n = -2 \) resonance: (a) Electron pitch angle \( \theta_\parallel \) at the equilibrium \( (\hat{q} = 0) \) and critical pitch angle \( \theta_n (\hat{q} = 0) \) vs the normalized inductive electric field \( (E_\perp = 3000 \text{ V/m}) \). (b) Critical pitch angle \( \theta_\parallel \) vs perpendicular wave electric field \( E_\perp \) for wave and plasma conditions given in previous figures. Note at high \( E_\perp \) values the dependence \( \theta_\parallel \approx E_\perp^{3/2} \).
an efficient pitch angle scattering process is found at $\gamma_n \approx 17$ ($\sim 8$ MeV). Nevertheless, the equilibrium is reached at $\theta_n \approx 0.24$ rad, for which $k_L \rho_L \approx 5$, so that, for a complete description of the process, the quasi-perpendicular Landau interaction between the wave and the particle should be included when the electron approaches the equilibrium point.

For the $n = -2$ resonance, the conditions for an efficient interaction appear to be quite different due to the linear dependence of the diffusion coefficient, $D_{pV}$, on the electron pitch angle [in contrast to the dependence $D_{pV} \propto (k_L \rho_L)^{-1/2}$ for the $n = -1$ resonance]; According to Eqs. (16) and (17), for $n = -2$, $D_{pV} \propto J_2(k_L \rho_L)^{3/2}(k_L \rho_L)^2$ so that, using the assumption of small Larmor radius $(k_L \rho_L \ll 1)$ and that $J_2(x) \approx x^3/2^2e^x$ for $x \ll 1$, we would get $D_{pV} \propto k_L \rho_L \propto k_L \gamma \sin \theta \Omega_0$ where it has been used that $\rho_L = v_L / \omega e$, $\omega_c = \Omega_0 / \gamma$ and $v_L \approx c \sin \theta$. Thus, if the electron pitch angle is not high enough when the particle reaches the resonant region, the interaction $(D_{pV})$ will be small and the electron will cross the resonance ($\dot{q}_n > 0$). This leads to a minimum pitch angle needed for an efficient wave–particle interaction, determined by the condition $\dot{q}_n = 0$, and that will be called the critical pitch angle, $\theta_c$, for efficient interaction. Such critical pitch angle $(\theta_c = \theta_c)$ is plotted in Fig. 3(a) vs $D$ together with the equilibrium pitch angle $\theta_1$ for $E = 3000$ V/m: The energy blocking at the resonance (only occurring, as explained above, if the condition $\dot{q}_n = 0$ is found before $\dot{q}_n > 0$) will take place for electrons entering the resonant region with a pitch angle $\theta$ larger than $\theta_c = \theta_1$. On the other hand, as in the case $n = -1$, the mechanism will not be effective if the inductive electric field accelerating the runaway electrons is large enough: the upper electric field $E_{b}$ for an efficient resonant interaction, as illustrated in Fig. 3(a), is now determined by the intersection of the curves $D(\theta_1)$ and $D(\theta_c)$.

The dependence of the critical pitch angle $\theta_c$ on $E_1$ is shown in Fig. 3(b): $\theta_c$ decreases for increasing $E_1$ as, for larger values of the perpendicular wave field, the diffusion coefficient $D_{pV} \propto E_{1}^{3/2}$ increases, so that smaller pitch angle values are required for an efficient interaction. Using the condition $\dot{q}_n = 0$, the following analytical formula for $\theta_c$, valid for $k_L \rho_L \ll 1$, can be found:

$$\sin \theta_c = -A_n + [A_n^2 + B_n]^{1/2},$$

(25)

$$A_n = \frac{k_L \Omega_0 D_{\text{LH}}}{8 k_L^2 c F_{sy} \gamma_n^2} \left( \frac{\gamma_n}{2} \right)^{3/2} \frac{k_L \gamma}{2 \Omega_0},$$

$$B_n = \frac{1}{F_{sy} \gamma_n^2} \left( D - 1 - \frac{\alpha}{\gamma_n} - F_{sy} \gamma_n^4 \right),$$

(26)

and $\gamma_n$ is the energy at the $n = -2$ resonance, $\gamma_n \approx 2 \Omega_0 / (k_L c - \omega)$. In the limit of high $E_1$, this relation leads to $\theta_c \approx \sin \theta_c = B \Omega_0 / 2 \Omega_0$ and, therefore, $\theta_c \propto 1/E_{1}^{3/2}$, as illustrated in Fig. 3(b). A detailed discussion on the conditions under which these analytical approximations for $\theta_c$ have been derived can be found in Appendix C.

IV. CONCLUSIONS

In this paper, a diffusion coefficient has been derived to describe the interaction in momentum space between runaway electrons and lower hybrid waves in a cyclotron resonance. This effect has been included in a single particle description of the runaway dynamics to evaluate the efficiency of such mechanism to increase the electron pitch angle and therefore the synchrotron radiation power to a high enough level to block the runaway energy. The conditions for an effective interaction at the resonance have been established, and it has been shown that the fast pitch angle scattering event observed in TEXTOR could be due, as suggested in Refs. 7 and 8, to the interaction at the $n = -1$ resonance with lower hybrid waves excited via the Parail Pogutse instability if the perpendicular wave-field is larger than $\sim 300$ V/m. The analysis has also shown that, for typical lower hybrid current drive conditions $(N_1 \approx 2, N_\perp / \Omega_0 \approx 10, f = 2 - 10$ GHz), an effective pitch angle scattering process can be found for the runaway electrons due to the resonant coupling with the waves via anomalous Doppler broadening at the $n = -1$ resonance. Nevertheless, two considerations must be made: (1) Even for $n = -1$, under typical current drive conditions, when the particle approaches to the equilibrium point during the scattering process, quasi-perpendicular Landau interaction $(k_L \rho_L > 1)$ may take place, which should be considered for a complete description of the electron dynamics; (2) during current drive experiments, the accelerating electric field $E_{b}$ is decreased (ideally, for full current drive, should go to zero). This reduction of the electric field $E_{b}$ leads to a rapid fall of the runaway energy, preventing the resonant interaction between the electrons and the wave. Thus, for the experiments carried out in the Frascati Tokamak Upgrade (FTU) $^{12}$ $(N_1 = 1.8, f = 8$ GHz, $B_0 = 5.5$ T, $\gamma_n = 24$ ($\sim 12$ MeV) for $n = -1$, the decrease of the electron energy observed during LH power injection has been proved to be associated with the reduction of the loop voltage $(E_{b})$ during the current drive phase, with no evidence for a resonant wave–particle interaction at MeV electron energies.

It has also been proposed $^{5,7}$ that, with this new mechanism, lower hybrid waves under proper conditions can be used to disrupt runaway electron beams in situations in which the runaway energy should be controlled for safer tokamak operation. This is particularly important for disruption generated runaway electrons. However, some care must be taken as, during disruptions, the plasma density is large due to the impurity influx from the overheated wall structures, so that the accessibility of lower hybrid waves externally applied will be quite restricted. On the other hand, given the large electric fields induced during the disruption, the equilibrium at the resonance could take place at high values of the pitch angle (which, as shown in Figs. 2 and 3, is an increasing function of the inductive electric field). Thus, after the initial pitch angle scattering process at the resonance via anomalous Doppler broadening, nearly perpendicular Landau interaction...
between the wave and the particle \((k \cdot \rho_L > 1)\) would occur when the particle is close to the equilibrium, which should be taken into account for a full description of the interaction at the resonance. As an example, Figs. 4(a) and 4(b) show, respectively, the pitch angle \(\theta_n\) at the resonant equilibrium \(n = -1\) and the parameter \(\eta = k \cdot \rho_L\) as function of the normalized inductive electric field, calculated for standard wave parameters during lower hybrid current drive \(f = 3.7 \text{ GHz}, N_\parallel = 2\) and typical plasma conditions during a disruption in JET \(B_0 = 3 \text{ T}, R_0 = 3 \text{ m}, n_e = 10^{20} \text{ m}^{-3}, T_e = 5 \text{ eV}, Z_{eff} = 3\).

It is observed that, for typical values of the electric field during the disruption \(\sim 10^3 \text{ V/m} (D = 110 \text{–} 1100)\), the pitch angle \(\theta_n\) is large \((\theta_n > 0.8 \text{ rad})\) and the parameter \(\eta > 50\).

ACKNOWLEDGMENT

This work was done under financial support from Dirección General de Investigación Project No. FTN2000-0965.

APPENDIX A: QUASILINEAR DIFFUSION COEFFICIENT

1. Diffusion coefficient

We will derive the diffusion coefficient for the gyromomentum describing the interaction with the LH waves in the cyclotron resonance.

From Eq. (11), the change \(\delta p_\perp\) in gyromomentum between 0 and \(\delta t\), by integration, will be given by

\[
\delta p_\perp = -\frac{e E_\perp \omega_c n J_n(k \cdot \rho_L)}{k \cdot v_\perp} \sin \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t + \Phi}{\sin \Phi} - \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega
\]

\[
= 2 \frac{e E_\perp \omega_c n}{k \cdot v_\perp} J_n(k \cdot \rho_L) \times \cos \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t + \Phi}{2} \times \sin \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t}{2}.
\]

(A1)

Averaging over the initial phase \(\Phi\)

\[
\langle \delta p_\perp (\delta t) \rangle = \frac{1}{2 \pi} \int_0^{2\pi} \frac{2 e E_\perp \omega_c n}{k \cdot v_\perp} \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right]^2 \times \cos^2 \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t + \Phi}{2} \times \sin^2 \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t}{2} d\Phi.
\]

(A2)

and using the identity, \(\nu \sin^2(\nu) / \pi \nu^2 = \delta(\nu)\), if \(\nu = 0\), we get

\[
\lim_{\delta t \to \infty} \sin^2 \left[ \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right] \frac{\delta t}{2} = \frac{\pi \delta t}{2} \delta \left( \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right).
\]

(A3)

Therefore, we will have

\[
\lim_{\delta t \to \infty} \left\langle \frac{\delta p_\perp (\delta t) \delta p_\perp (\delta t)}{\delta t} \right\rangle = \frac{\pi}{2} \delta \left( \frac{n \Omega_0}{\gamma} + k \cdot \mathbf{v} \cdot \omega \right).
\]

(A4)
2. Resonance width

Finally, to get the quasilinear diffusion coefficient, an averaging must be made over the resonance width.

The resonance condition (10) can be expressed as \( w-w_s=0 \), where \( w=n\omega+\omega c_k||p||/m_c\gamma \), and \( w_s=\omega \). Hence, from Eq. (14), we get

\[
\dot{w} = -\Delta \cos \delta, \tag{A5}
\]

with

\[
\Delta = \frac{eE||k||J_n(k ||p||)}{m_c\gamma} \left(1 + \frac{n}{k ||p|| E||p||} \right)
\]

and

\[
\delta = w - w_s. \tag{A7}
\]

Then, the resonance width for the set of differential equations (A5) and (A7) will be given by

\[
\Delta w = 2\sqrt{\Delta}. \tag{A8}
\]

Therefore, we will have for the diffusion coefficient

\[
D_{p_s||p_s||} = \frac{1}{2\Delta w} \int_{w_s-\Delta w}^{w_s+\Delta w} \frac{eE||k||\omega_c c_s J_n(k ||p||)}{k ||p||} \left[\frac{1}{2w} \left(1 + \frac{n}{k ||p|| E||p||} \right) \right] \cdot \delta \cdot (w-w_s) \, dw
\]

where \( \omega_{c_s} \) is the value of the electron cyclotron frequency at the resonance, \( n \omega_{c_s} = \omega - k ||p|| \), so that

\[
D_{p_s||p_s||} = \frac{\pi}{2\Delta w} \left(1 + \frac{n}{k ||p|| E||p||} \right)^2 \left(1 + \frac{n}{k ||p|| E||p||} \right)^2. \tag{A9}
\]

APPENDIX B: CALCULATION OF THE UPPER ELECTRIC FIELD \( D_{up} \) FOR THE \( n=-1 \) RESONANCE

For the \( n=-1 \) resonance, the upper electric field \( D_{up} \) is given by the minimum of the curve \( D(\theta) \). The curve \( D(\theta) \) is determined setting \( \dot{q}||=0 \) in Eq. (19) yielding, assuming \( \gamma_n>1 \) \( (u \sim c) \) and \( \cos \theta=1 \) (so that \( q||=q=\gamma_n \))

\[
D \simeq 1 + \frac{\alpha}{\gamma_n} + F_{gy} \gamma_n^4 + F_{gy} \gamma_n^2 \sin^2 \theta
\]

\[
+ \frac{D_{LH} k ||p||}{2|n|k ||p||} \left( \frac{k ||p||}{2\Omega_0} \right) \frac{\left(\sin \theta \right)^{3/2}}{\left(\left|\Omega_0\right| \right)^{3/2}} - \frac{3}{2}, \tag{B2}
\]

which, for \( n=1 \), leads to

\[
D \simeq 1 + \frac{\alpha}{\gamma_n} + F_{gy} \gamma_n^4 + F_{gy} \gamma_n^2 \sin^2 \theta
\]

\[
+ \frac{D_{LH} k ||p||}{2\Omega_0} \left( \frac{k ||p||}{2\Omega_0} \right) \frac{\left(\sin \theta \right)^{3/2}}{\left(\left|\Omega_0\right| \right)^{3/2}} - \frac{3}{2}, \tag{B3}
\]

with \( x=(\sin \theta)^{1/2} \) and \( \gamma_n=\Omega_0/(k ||p||-\omega) \).

The minimum of \( D \) vs \( \theta \) in (B3) can be found from the condition \( dD/d\theta=0 \) so that we obtain

\[
0 = -\frac{D_{LH} k ||p||}{2\Omega_0} \left( \frac{k ||p||}{2\Omega_0} \right) \frac{3}{2} x^2 + 4F_{gy} \gamma_n^2 x^3 \tag{B4}
\]

and

\[
x=(\sin \theta)^{1/2} \left[ \frac{k ||D_{LH} \Omega_0}{8k ||p|| F_{gy} \gamma_n^2} \right]^{1/5} \tag{B5}
\]

Substituting in (B3), we get

\[
D_{up} \simeq 1 + \frac{\alpha}{\gamma_n} + F_{gy} \gamma_n^4 + 5F_{gy} \gamma_n^2 \sin^2 \theta
\]

\[
+ \frac{k ||D_{LH} \Omega_0}{8k ||p|| F_{gy} \gamma_n^2} \left( \frac{k ||p||}{2\Omega_0} \right)^{1/5}. \tag{B6}
\]

In this expression, the relation \( \gamma \) for \( D_{LH} \) must be used, where the coefficient \( A_{LH} \) \[Eq. (22)\] can be simplified to \( (u \sim c, n \omega_{c_s}=-\omega-k ||p||, E||p||-k ||p||) A_{LH} \approx (1+1/N^2)^{1/2} \). Hence, we can finally write

\[
D_{up} \simeq 1 + \frac{\alpha}{\gamma_n} + F_{gy} \gamma_n^4 + 5F_{gy} \gamma_n^2 \sin^2 \theta
\]

\[
\times \frac{\pi \Omega_0}{32A_{LH} n ||p||} \left( \frac{k ||p||}{2\Omega_0} \right)^{1/5} a^{4/5} a^{6/5}. \tag{B7}
\]

APPENDIX C: CALCULATION OF THE CRITICAL PITCH ANGLE \( \theta_c \) FOR THE \( n=-2 \) RESONANCE

The critical pitch angle \( \theta_c \) for efficient wave–particle coupling \( (n=-2) \) is determined by the condition \( \dot{q}||=0 \) \( (\theta_c=\theta) \). The relation \( D(\theta) \) \( (\dot{q}||=0) \), derived under the assumption of small Larmor radius, is given by Eq. (B2) for the \( n \) cyclotron resonance and yields for the case \( n=-2 \)

\[
D \simeq 1 + \frac{\alpha}{\gamma_n} + F_{gy} \gamma_n^4 + F_{gy} \gamma_n^2 \sin^2 \theta
\]

\[
+ \frac{k ||D_{LH} \Omega_0}{4k ||p|| \Omega_0^2} \left( \frac{k ||p||}{2\Omega_0} \right) \frac{\left(\sin \theta \right)^{3/2}}{\left(\left|\Omega_0\right| \right)^{3/2}} - \frac{3}{2}, \tag{C1}
\]

where it has been used that, for \( n=-2 \), \( \theta_c=\theta \), and \( \gamma_n=\Omega_0/(k ||p||-\omega) \).

Equation (C1) constitutes a second-order equation in \( x=\sin \theta \).
\[ x^2 + 2A_n x - B_n = 0, \]  

with
\[
A_n = \frac{k_{||} \Omega_0 D_{\text{LH}}}{8 k \gamma_n D_{g_y} \gamma_n^2} \left( \frac{\gamma_n}{2} \right)^{3/2} \left( \frac{k_{||} \gamma_n}{2 \Omega_0} \right)^{3/2},
\]
\[
B_n = \frac{1}{F_{g_y} \gamma_n} \left( D - 1 - \frac{\alpha}{\gamma_n} - F_{g_y} \gamma_n^4 \right).
\]

from which the critical pitch angle can be obtained
\[ x = \sin \theta_c = -A_n + \left[ A_n^2 + B_n \right]^{1/2}. \]  

For high enough values of \( E_\perp \), \( D_{\text{LH}} \) will be large and Eq. (C3) can be written
\[ \theta_c = \sin \theta_c \approx \frac{B_n}{2A_n} \left( \frac{4 k_{||}^2 \gamma_n}{k_{||} D_{\text{LH}} \Omega_0} \right)^{3/2} \left( \frac{2 \Omega_0}{k_{||} \gamma_n} \right)^{3/2} \left( D - 1 - \alpha \gamma_n - F_{g_y} \gamma_n^4 \right), \]  

and using the relation (21) for \( D_{\text{LH}} \), and the approximation \( A_{\text{LH}} \approx (1 + 1/N_{||}^{1/2})^{1/2} \), we would get
\[ \theta_c \approx \frac{4 \nu_c c^3}{\pi \Omega_0 E_{||}^{3/2}} \left( \frac{2 m_{k_{||}}}{e \gamma_n} \left( 1 + \frac{1}{N_{||}} \right)^{1/3} \right)^{3/2} \times \left( \frac{2 \Omega_0}{k_{||} c} \right)^{3/2} \left( D - 1 - \frac{\alpha}{\gamma_n} - F_{g_y} \gamma_n^4 \right). \]  